Banking in an OLG Money Model

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Background and Roadmap

- A model that assigns a serious role for financial intermediation.
- Implications for general equilibrium macro.
- Implications for monetary policy.

Model: Environment

- Infinite time horizon, $t \in \mathbb{N}$.
- Agents are short-lived: two-period OLG, agent's age $j \in \{1, 2\}$.
- Two locations ("islands") of equal population size each period.
- Population size of young agents N_t on each island, obeys law

$$N_{t+1} = \eta N_t, \qquad N_0 \text{ given.}$$

- Initial size N_0 old people given on each island.

Model: Preferences

- Agents face a location (equivalently, liquidity) shock at the end of $t = 1$.
- Each agent on island $i \in \{A, B\}$ faces probability $\pi \in (0, 1)$ of relocating to other island.
- By LLN for i.i.d. random variables, π is also the proportion of agents relocating from one island to the other.
- Agents don't value consumption when young. On island i old agents consume c_2^i.
- Ex-ante identical preference representations. E.g. agent on island A:

$$U(c_2^A, c_2^B) = (1 - \pi)u(c_2^A) + \pi u(c_2^B),$$

with $u' > 0$ and $u'' < 0$.

Model: Endowment and Technology

- Each young agent endowed with $y > 0$ units of good.
- Agent can invest k units of capital from given endowment, but prior to realization of random variable $i \in \{A, B\}$.
- Exists a linear storage technology on each island. Given k, $k \mapsto xk$.
- Ex-post, if relocation occurs, k is liquidated – i.e. Given k and if i is not the current island, then, $k \mapsto lk$.
- Assume $x > \eta > l > 0$.

First Best Allocation

- Planner knows proportion $(1-\pi)$ will stay and π relocate.
- Instructs all agents to invest $k=y$.
- For the proportion π $(A \rightarrow B)$ they are allocated the output xk from the investment k made by the same proportion π $(B \rightarrow A)$, and vice-versa.
- Each agent obtains perfect insurance against risk of capital liquidation. Consumption is smoothed over both states: $c_2^A = c_2^B = xk = xy$.
- This Pareto allocation is equivalent to a competitive equilibrium where agents can centrally exchange state-contingent claims to consumption in each state $i \in \{A, B\}$.

Information Friction

- Two frictions in the economic environment:
  - Limited communication: agents cannot communicate with others on the other island
  - Location/liquidity shock is private information
- Partial anonymity, private information and random relocation: prevent existence of complete securities and private contracts between agents.
- A ("serious") role/justification for emergence of financial intermediary: information and trading environment friction.

Autarky

- What if there is no benevolent social planner, nor markets for trading complete claims to risky consumption?
- This limit of the economy results in
  - agents investing all their endowment, so $k = y$ (since they don't value consumption when young);
  - consuming from hand to mouth conditional of where they are: $c_2^i = xk$ and $c_2^{!i} = lk$; and
  - Inefficiency: output loss on each island is $\pi N_t (x - l) y$.
- Note: $i \in \{A, B\}$ and $!i := \neg(i)$ (Read: "not i").

Money and Banking

- Note: $x > \eta$ shuts down usual Pareto improving role of fiat money in OLG model.
- Focus is on role of fiat money in overcoming information friction:
  - As store of value across locations
  - As means of economizing on effect of liquidity/location shock (scrapping of projects).
- So now consider economy with fiat money and banking.

Money and Banking

Money:

- Introduced by government fiat. Initial money stock M_0 held by initial old.
- At time t, total stock of money is M_t
- Government injects new money $(z-1)M_{t-1}$ at time t,
- New money is lump-sum transferred to each young as τ, prior to agents knowing their (private) liquidity shock.
- Value of money, $v_t = 1/P_t$, is inverse of price level of consumption good.
- Accounting:

$$N_t \tau = (z-1) v_t M_{t-1} \tag{GBC}$$

Money and Banking

Banking:

- Suppose emergence of intermediaries offering young a deposit/insurance contract.
- Contract stipulates:
  - Young agents to hold a security exchangeable for cash, on demand.
  - Young agents assigns right to bank to deposited claims on their endowments.
  - If agent's right to liquidity is not executed, the security pays off the competitive return x.

Money and Banking

With money and banking now:

- Agents that realize a relocation shock can and will exercise the option to withdraw from bank;
- carry cash to new location.
- Bank on island i anticipates this: need to carry enough (real) cash reserves q to meet expected withdrawals:

$$\pi c_2^{!i} \leq q \frac{P_t}{P_{t+1}} \equiv q \frac{v_{t+1}}{v_t} \tag{LC}$$

c.f. In autarky, investment capital is sunk and not portable. Agent relocating must liquidate and carry discounted amount of good lk with them.

Money and Banking

- Perfect competitition and free entry implies bank earns zero profit in equilibrium.
- Equivalent to a competitive bank maximizing expected utility of its representative depositor.

Money and Banking

Agent on island i has E.U.: $U(c_2^i, c_2^{!i}) = (1-\pi)u(c_2^i) + \pi u(c_2^{!i})$.

Bank on island A solves problem (symmetrically on island B):

$$\max U(c_2^i, c_2^{!i}) \tag{P1}$$

subject to

$$q + k \leq y + \tau \qquad \text{(Feasibility, F)}$$

$$(1-\pi)c_2^i + \pi c_2^{!i} \leq xk + q\frac{P_t}{P_{t+1}} \qquad \text{(Balance Sheet, BS)}$$

$$\pi c_2^{!i} \leq q\frac{P_t}{P_{t+1}} \qquad \text{(Liquidity constraint, LC)}$$

Money and Banking

If $x > P_t/P_{t+1}$,

- there is positive inflation rate; equiv. real return on long-term asset dominates real return on holding (real) cash reserves.
- Optimizing bank will want to hold as little cash reserve as possible in this case.
- So at optimum, bank will choose q such that liquidity constraint binds: $\pi c_2^{!i} = q\frac{P_t}{P_{t+1}}$.
- Conversely, bank wants to invest as much k as possible: so (Feasibility) and (BS) bind.

Money and Banking

An interior optimum is characterized by

$$\frac{u'(c_2^i)}{u'(c_2^{!i})} = \frac{P_t}{P_{t+1}} \frac{1}{x}$$

and (F), (BS), and (LC) binding.

- When inflation rate is positive $x > P_t/P_{t+1}$, bank optimally chooses (q, k) such that $MRS(c_2^i, c_2^{!i})$ equals the marginal rate of transformation under the long-term technology's payoff $\frac{1}{x}$, no?
- Then?

Money and Banking

- Inflation tax wedge: P_t/P_{t+1}.
- Note: as long as $x \neq P_t/P_{t+1}$, equilibrium under money and banking is not efficient: $c_2^i \neq c_2^{!i}$.
- i.e. banking mechanism does not completely smooth agents' consumption allocation across both states, c.f. Pareto allocation.

Exercise

Prove that if $x > P_t/P_{t+1}$, then $c_2^i > c_2^{!i}$, and therefore an inefficient allocation obtains. Why?

Monetary Equilibrium

Definition

A (stationary) equilibrium with money and banking is an allocation (q, k, c_2^A, c_2^B) satisfying

1. $v_{t+1}/v_t = \eta/z$
2. Money market clearing:

$$v_t M_t = N_t q \tag{MM}$$

3. Government budget constraint holds:

$$N_t \tau = (z-1) v_t M_{t-1} \tag{GBC}$$

4. Consumers and banks optimize: (P1) s.t. (F), (BS), (LC).

Monetary Equilibrium

Let's derive the implication of this:

- From (GBC) and (MM) we get

$$\tau = \frac{z-1}{z} q^d(\eta/z, \tau)$$

  where $q^d(\eta/z, \tau)$ is equilibrium demand for liquidity.
- This expression encodes the equilibrium purchasing power of money as a function of the policy parameter z.
- So implicitly, $\tau = \tau(z)$, is an equilibrium function of z.

Monetary Equilibrium

- From binding (LC) equilibrium consumption of relocators are:

$$c_2^{!i} = \frac{1}{\pi}\left(\frac{\eta}{z}\right) q^d(\eta/z, \tau(z)).$$

where $q^d(\eta/z, \tau(z))$ is equilibrium demand for liquidity, a best-response to policy z.

- Given binding (LS), from (F) and (BS), along with (GBC) and (MM), consumption of non-relocators are:

$$c_2^{i} = \frac{x}{1-\pi}\left[y - \frac{q^d(\eta/z, \tau(z))}{z}\right].$$

Monetary Equilibrium: Implications

Implications: for a given monetary policy z, between any t and $t+1$,

- Stationary equilibrium return on money, is η/z
- Return on premature liquidation of project is l
- If $\eta/z > l$, individuals who get liquidity/relocation shock simply withdraw from bank. So old agents are better off.
- Young do bear a cost in this monetary setup. Ex post some of young's resources are redistributed to benefit the relocating old
- If cost of scrapping is sufficient high (l sufficiently low) the young willing to tolerate this redistribution effect of the money and banking system

Monetary Equilibrium: Implications

More implications:

- $q^d(\eta/n, \tau(z))/y$ is model's equilibrium *reserve-deposit* ratio.
- In the data, this ratio is inversely related to nominal interest rate (hence, inflation; Fisher relation).
- If $\partial q^d(\eta/n, \tau(z))/\partial z < 0$, then model equilibrium consistent with this empirical observation.
- High (low) inflation (z) implies high (low) opportunity cost of liquidity reserve demand, hence low (high) q^d.

Monetary Equilibrium: Implications

More implications:

- Model's real GDP is

$$Y_t = \left\{ y + \frac{x}{\eta} \left[y - q^d(\eta/n, \tau(z)) \right] \right\} N_t.$$

- Note: if $\partial q^d(\eta/n, \tau(z))/\partial z < 0$, then $\partial Y_t/\partial z > 0$.
- A story to account for liquidity trap (low/zero nominal interest rate and low output): Japan since the 1990's and Great Depression U.S. economy.

Monetary Equilibrium: Implications

In words:

- Low (high) inflation decreases (increases) opportunity cost of holding cash (ex post).
- Ex ante, banks anticipate this, so substitute investment towards (away from) cash reserves.

An optimal monetary policy

- Suppose a monetary policy maker (Wilbur McMuffin) seeks to maximize young agent's ex ante welfare.
- Denote as "WM" for welfare-maximizing Monetary-policy maker.
- Unlike omni-α, $\alpha \in \{$scient, present, potent$\}$, benevolent planner, WM is restricted to an indirect and finite number of policy instruments.
- i.e. WM cannot tell people what to do directly.

An optimal monetary policy

- WM's welfare criterion is

$$W(z) = (1-\pi)u\left(\frac{x(y - z^{-1}q^d(\eta/n, \tau(z)))}{1-\pi}\right) + \pi u\left(\frac{\eta z^{-1} q^d(\eta/n, \tau(z))}{\pi}\right).$$

An optimal monetary policy

- An optimal monetary policy w.r.t. to instrument z satisfies

$$\frac{u'\left(\frac{x(y-z^{-1}q^d(\eta/n,\tau(z)))}{1-\pi}\right)}{u'\left(\frac{\eta z^{-1}q^d(\eta/n,\tau(z))}{\pi}\right)}=\frac{\eta}{x}.$$

- For an arbitrary fixed z the equilibrium allocation is given by:

$$\frac{u'\left(\frac{x(y-z^{-1}q^d(\eta/n,\tau(z)))}{1-\pi}\right)}{u'\left(\frac{\eta z^{-1}q^d(\eta/n,\tau(z))}{\pi}\right)}=\frac{\eta}{zx}.$$

- Hence, if $z=1$, the policy is also an ex-ante optimal policy.

An optimal monetary policy

- What does $z = 1$ mean? Optimal policy is to hold money supply constant forever.
- At any $z < 1$ (deflationary policy, e.g. including Friedman rule $z = \eta/x < 1$) marginal benefit of money as liquidity shock insurance not enough to compensate marginal cost of diminished investment (hence future output/consumption).
- At any $z > 1$ (inflationary policy) marginal benefit of money as liquidity shock insurance not enough to compensate marginal cost of diminished investment (hence future output/consumption).
- At $z = 1$ allocation is (constrained) optimal. WM still cannot undo market friction's effect on imperfect insurance contract offered by bank. Intuition: 2 frictions, 1 policy instrument.

Fully optimal monetary policy

In principle, why is WM restricted to just one instrument z?

- WM is a national animal – spanning both islands. Banks are local. WM can also act as bank on both islands. Undo spatial and information friction.
- How to operationalize? Add another instrument: Issue money backed by private claims. Options:
  - Behave like the commercial banks. Make collateralized loans.
  - Engage in OMO: swap cash for private securities/claims to any island's projects.
  - Open a discount window that discounts commercial paper.
- Two instruments to undo two frictions. Attain first-best or efficient allocation. WM now mimicks the planner.

Conclusion

Positive:

- Model with two frictions: information and spatial (liquidity shock and missing private-claims markets) friction.
- Trading environment frictions imply that in absence of money (pure role of providing liquidity insurance) and banking (aggregator of individual risk), autarky is not efficient.
- Money and banking – does better than autarky, but does not provide perfect insurance of individual consumption risks.
- Monetary equilibrium with banking can account for some simple facts. What are they?

Conclusion

Normative:

- Optimal monetary policy with restricted policy instrument (z) trade-offs benefit-vs-cost of inflation on money-vs-investment. But still subject to limited insurance environment (bank).
- Hypothetically, fully optimal policy requires WM to internalize spatial frictions effect on imperfect liquidity insurance provided by banks. This requires another instrument.