Banking in an OLG Money Model

Timothy Kam

Research School of Economics
Australian National University

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Outline

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Background and Roadmap

A model that assigns a serious role for financial intermediation.

Implications for general equilibrium macro.

Implications for monetary policy.
Model: Environment

- Infinite time horizon, \( t \in \mathbb{N} \).
- Agents are short-lived: two-period OLG, agent’s age \( j \in \{1, 2\} \).
- Two locations ("islands") of equal population size each period.
- Population size of young agents \( N_t \) on each island, obeys law
  \[
  N_{t+1} = \eta N_t, \quad N_0 \text{ given}.
  \]
- Initial size \( N_0 \) old people given on each island.
Model: Preferences

- Agents face a location (equivalently, liquidity) shock at the end of $t = 1$.
- Each agent on island $i \in \{A, B\}$ faces probability $\pi \in (0, 1)$ of relocating to other island.
- By LLN for i.i.d. random variables, $\pi$ is also the proportion of agents relocating from one island to the other.
- Agents don’t value consumption when young. On island $i$ old agents consume $c_2^i$.
- Ex-ante identical preference representations. E.g. agent on island A:

$$U(c_2^A, c_2^B) = (1 - \pi)u(c_2^A) + \pi u(c_2^B),$$

with $u' > 0$ and $u'' < 0$. 
Model: Endowment and Technology

- Each young agent endowed with \( y > 0 \) units of good.
- Agent can invest \( k \) units of capital from given endowment, but prior to realization of random variable \( i \in \{A, B\} \).
- Exists a linear storage technology on each island. Given \( k \), \( k \mapsto xk \).
- Ex-post, if relocation occurs, \( k \) is liquidated – i.e. Given \( k \) and if \( i \) is not the current island, then, \( k \mapsto lk \).
- Assume \( x > \eta > l > 0 \).
First Best Allocation

- Planner knows proportion \((1 - \pi)\) will stay and \(\pi\) relocate.
- Instructs all agents to invest \(k = y\).
- For the proportion \(\pi (A \rightarrow B)\) they are allocated the output \(xk\) from the investment \(k\) made by the same proportion \(\pi (B \rightarrow A)\), and vice-versa.
- Each agent obtains perfect insurance against risk of capital liquidation. Consumption is smoothed over both states: \(c^A_2 = c^B_2 = xk = xy\).
- This Pareto allocation is equivalent to a competitive equilibrium where agents can centrally exchange state-contingent claims to consumption in each state \(i \in \{A, B\}\).
Information Friction

- Two frictions in the economic environment:
  - Limited communication: agents cannot communicate with others on the other island
  - Location/liquidity shock is private information

- Partial anonymity, private information and random relocation: prevent existence of complete securities and private contracts between agents.

- A ("serious") role/justification for emergence of financial intermediary: information and trading environment friction.
Autarky

What if there is no benevolent social planner, nor markets for trading complete claims to risky consumption?

This limit of the economy results in

- agents investing all their endowment, so $k = y$ (since they don't value consumption when young);
- consuming from hand to mouth conditional of where they are: $c_{2}^{i} = x k$ and $c_{2}^{i} = lk$; and
- Inefficiency: output loss on each island is $\pi N_{t} (x - l)y$.

Note: $i \in \{A, B\}$ and $!i := \neg(i)$ (Read: “not $i$”).
Money and Banking

- Note: $x > \eta$ shuts down usual Pareto improving role of fiat money in OLG model.
- Focus is on role of fiat money in overcoming information friction:
  - As store of value across locations
  - As means of economizing on effect of liquidity/location shock (scrapping of projects).
- So now consider economy with fiat money and banking.
Money and Banking

Money:
- Introduced by government fiat. Initial money stock $M_0$ held by initial old.
- At time $t$, total stock of money is $M_t$
- Government injects new money $(z - 1)M_{t-1}$ at time $t$,
- New money is lump-sum transferred to each young as $\tau$, prior to agents knowing their (private) liquidity shock.
- Value of money, $v_t = 1/P_t$, is inverse of price level of consumption good.
- Accounting:

$$N_t \tau = (z - 1)v_t M_{t-1} \quad \text{(GBC)}$$
Money and Banking

Banking:

- Suppose emergence of intermediaries offering young a deposit/insurance contract.
- Contract stipulates:
  - Young agents to hold a security exchangeable for cash, on demand.
  - Young agents assigns right to bank to deposited claims on their endowments.
  - If agent’s right to liquidity is not executed, the security pays off the competitive return $x$. 
Money and Banking

With money and banking now:

- Agents that realize a relocation shock can and will exercise the option to withdraw from bank;
- carry cash to new location.
- Bank on island $i$ anticipates this: need to carry enough (real) cash reserves $q$ to meet expected withdrawals:

$$
\pi c_2^{i} \leq q \frac{P_t}{P_{t+1}} \equiv q \frac{v_{t+1}}{v_t}
$$

(LC)

c.f. In autarky, investment capital is sunk and not portable. Agent relocating must liquidate and carry discounted amount of good $lk$ with them.
Money and Banking

- Perfect competition and free entry implies bank earns zero profit in equilibrium.
- Equivalent to a competitive bank maximizing expected utility of its representative depositor.
Money and Banking

Agent on island $i$ has E.U.: \[ U(c_2^i, c_2^{l,i}) = (1 - \pi)u(c_2^i) + \pi u(c_2^{l,i}). \]

Bank on island A solves problem (symmetrically on island B):

\[
\begin{align*}
\max & \ U(c_2^i, c_2^{l,i}) \\
\text{subject to} & \quad q + k \leq y + \tau \\
& \quad (1 - \pi)c_2^i + \pi c_2^{l,i} \leq xk + q \frac{P_t}{P_{t+1}} \\
& \quad \pi c_2^{l,i} \leq q \frac{P_t}{P_{t+1}}
\end{align*}
\]

(Feasibility, F) \hspace{1cm} (Balance Sheet, BS) \hspace{1cm} (Liquidity constraint, LC)
If $x > \frac{P_t}{P_{t+1}}$,

- there is positive inflation rate; equiv. real return on long-term asset dominates real return on holding (real) cash reserves.
- Optimizing bank will want to hold as little cash reserve as possible in this case.
- So at optimum, bank will choose $q$ such that liquidity constraint binds: $\pi c_2^i = q \frac{P_t}{P_{t+1}}$.
- Conversely, bank wants to invest as much $k$ as possible: so (Feasibility) and (BS) bind.
Money and Banking

An interior optimum is characterized by

\[
\frac{u'(c_2^i)}{u'(c_2^{li})} = \frac{P_t}{P_{t+1}} \frac{1}{x}
\]

and (F), (BS), and (LC) binding.

- When inflation rate is positive \( x > P_t/P_{t+1} \), bank optimally chooses \((q, k)\) such that \( MRS(c_2^i, c_2^{li}) \) equals the marginal rate of transformation under the long-term technology’s payoff \( \frac{1}{x} \), no?
- Then?
Money and Banking

- Inflation tax wedge: $\frac{P_t}{P_{t+1}}$.
- Note: as long as $x \neq \frac{P_t}{P_{t+1}}$, equilibrium under money and banking is not efficient: $c^i_2 \neq c^l_2$.
- i.e. banking mechanism does not completely smooth agents’ consumption allocation across both states, c.f. Pareto allocation.

Exercise

Prove that if $x > \frac{P_t}{P_{t+1}}$, then $c^i_2 > c^l_2$, and therefore an inefficient allocation obtains. Why?
Monetary Equilibrium

Definition

A (stationary) equilibrium with money and banking is an allocation \((q, k, c^A_2, c^B_2)\) satisfying

1. \(v_{t+1}/v_t = \eta/z\)
2. Money market clearing:
   \[v_t M_t = N_t q\] \hspace{1cm} (MM)
3. Government budget constraint holds:
   \[N_t \tau = (z - 1)v_t M_{t-1}\] \hspace{1cm} (GBC)
4. Consumers and banks optimize: (P1) s.t. (F), (BS), (LC).
Let’s derive the implication of this:

- From (GBC) and (MM) we get

\[ \tau = \frac{z - 1}{z} q^d(\eta/z, \tau) \]

where \( q^d(\eta/z, \tau) \) is equilibrium demand for liquidity.

- This expression encodes the equilibrium purchasing power of money as a function of the policy parameter \( z \).

- So implicitly, \( \tau = \tau(z) \), is an equilibrium function of \( z \).
Monetary Equilibrium

• From binding (LC) equilibrium consumption of relocators are:

\[ c^i_2 = \frac{1}{\pi} \left( \frac{\eta}{z} \right) q^d(\eta/z, \tau(z)). \]

where \( q^d(\eta/z, \tau(z)) \) is equilibrium demand for liquidity, a best-response to policy \( z \).

• Given binding (LS), from (F) and (BS), along with (GBC) and (MM), consumption of non-relocators are:

\[ c^*_2 = \frac{x}{1 - \pi} \left[ y - \frac{q^d(\eta/z, \tau(z))}{z} \right]. \]
Monetary Equilibrium: Implications

Implications: for a given monetary policy $z$, between any $t$ and $t + 1$,

- Stationary equilibrium return on money, is $\eta/z$
- Return on premature liquidation of project is $l$
- If $\eta/z > l$, individuals who get liquidity/relocation shock simply withdraw from bank. So old agents are better off.
- Young do bear a cost in this monetary setup. Ex post some of young’s resources are redistributed to benefit the relocating old
- If cost of scrapping is sufficient high ($l$ sufficiently low) the young willing to tolerate this redistribution effect of the money and banking system
Monetary Equilibrium: Implications

More implications:

- \( q^d(\eta/n, \tau(z))/y \) is model’s equilibrium reserve-deposit ratio.
- In the data, this ratio is inversely related to nominal interest rate (hence, inflation; Fisher relation).
- If \( \partial q^d(\eta/n, \tau(z))/\partial z < 0 \), then model equilibrium consistent with this empirical observation.
- High (low) inflation \( (z) \) implies high (low) opportunity cost of liquidity reserve demand, hence low (high) \( q^d \).
Monetary Equilibrium: Implications

More implications:

- Model’s real GDP is

\[ Y_t = \left\{ y + \frac{x}{\eta} \left[ y - q^d(\eta/n, \tau(z)) \right] \right\} N_t. \]

- Note: if \( \partial q^d(\eta/n, \tau(z))/\partial z < 0 \), then \( \partial Y_t/\partial z > 0 \).

- A story to account for liquidity trap (low/zero nominal interest rate and low output): Japan since the 1990’s and Great Depression U.S. economy.
Monetary Equilibrium: Implications

In words:

- Low (high) inflation decreases (increases) opportunity cost of holding cash (ex post).
- Ex ante, banks anticipate this, so substitute investment towards (away from) cash reserves.
An optimal monetary policy

- Suppose a monetary policy maker (Wilbur McMuffin) seeks to maximize young agent’s ex ante welfare.
- Denote as “WM” for welfare-maximizing Monetary-policy maker.
- Unlike omni-α, \( \alpha \in \{ \text{scient, present, potent} \} \), benevolent planner, WM is restricted to an indirect and finite number of policy instruments.
- i.e. WM cannot tell people what to do directly.
An optimal monetary policy

WM’s welfare criterion is

$$W(z) = (1 - \pi)u \left( \frac{x(y - z^{-1}q^d(\eta/n, \tau(z))))}{1 - \pi} \right)$$

$$+ \pi u \left( \frac{\eta z^{-1}q^d(\eta/n, \tau(z)))}{\pi} \right).$$
An optimal monetary policy

- An optimal monetary policy w.r.t. to instrument $z$ satisfies

\[
\frac{u'(x(y-z^{-1}q^d(\eta/n,\tau(z))))}{1-\pi} = \eta \frac{\eta}{x}.
\]

- For an arbitrary fixed $z$ the equilibrium allocation is given by:

\[
\frac{u'(x(y-z^{-1}q^d(\eta/n,\tau(z))))}{1-\pi} = \eta \frac{\eta}{zxx}.
\]

- Hence, if $z = 1$, the policy is also an ex-ante optimal policy.
An optimal monetary policy

- What does $z = 1$ mean? Optimal policy is to hold money supply constant forever.

- At any $z < 1$ (deflationary policy, e.g. including Friedman rule $z = \eta/x < 1$) marginal benefit of money as liquidity shock insurance not enough to compensate marginal cost of diminished investment (hence future output/consumption).

- At any $z > 1$ (inflationary policy) marginal benefit of money as liquidity shock insurance not enough to compensate marginal cost of diminished investment (hence future output/consumption).

- At $z = 1$ allocation is (constrained) optimal. WM still cannot undo market friction’s effect on imperfect insurance contract offered by bank. Intuition: 2 frictions, 1 policy instrument.
Fully optimal monetary policy

In principle, why is WM restricted to just one instrument? 

- WM is a national animal – spanning both islands. Banks are local. WM can also act as bank on both islands. Undo spatial and information friction.

- How to operationalize? Add another instrument: Issue money backed by private claims. Options:
  - Behave like the commercial banks. Make collateralized loans.
  - Engage in OMO: swap cash for private securities/claims to any island’s projects.
  - Open a discount window that discounts commercial paper.

- Two instruments to undo two frictions. Attain first-best or efficient allocation. WM now mimicks the planner.
Conclusion

Positive:

- Model with two frictions: information and spatial (liquidity shock and missing private-claims markets) friction.
- Trading environment frictions imply that in absence of money (pure role of providing liquidity insurance) and banking (aggregator of individual risk), autarky is not efficient.
- Money and banking – does better than autarky, but does not provide perfect insurance of individual consumption risks.
- Monetary equilibrium with banking can account for some simple facts. What are they?
Normative:

- Optimal monetary policy with restricted policy instrument ($z$) trade-offs benefit-vs-cost of inflation on money-vs-investment. But still subject to limited insurance environment (bank).

- Hypothetically, fully optimal policy requires WM to internalize spatial frictions effect on imperfect liquidity insurance provided by banks. This requires another instrument.