Volatility Shocks in Markets and Policies: What Matters for a Small Open Economy like Canada? *

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Abstract

We structurally estimate how much stochastic-volatility, relative to first-moment shocks, account for policy, demand or supply fluctuations in Canada. The historical Canadian business cycle is largely due to domestic technology and policy shocks. Time-varying volatilities dominate during times of turmoil and policy-environment changes. Our model-based shock-volatility accounting attributes the early 1980s crisis to international, cost-push, and monetary-policy shock volatilities; the adoption of inflation targeting in the early 1990s to monetary-policy uncertainty; the recessions around the early 1980s and 1990s to investment volatility; and income-tax and capital-gains tax reforms involve relatively large tax-policy uncertainty.

Keywords: Volatility Shocks; Stochastic Volatility; Small Open Economy; Internal vs. External

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1 Introduction

There has been a recent surge of interest in identifying and measuring time-variations in the riskiness of underlying shocks to a macroeconomy (Bloom et al., 2007; Bloom, 2009; Born and Pfeifer, 2014; Caggiano et al., 2014; Fernández-Villaverde et al., 2015; Baker et al., 2016). In this literature, riskiness is modelled in terms of stochastic volatility in postulated shocks to the economy—i.e., the second moments of the distributions of shock processes are stochastic. However, the analyses using structural general-equilibrium models with monetary and fiscal policies tend to focus on closed economies such as the U.S. In this paper, we focus on Canada as the case study for a developed small open economy. Our contribution is to structurally estimate how much stochastic-volatility, relative to first-moment shocks, account for fiscal- and monetary-policy, demand and supply sources of its business cycle.

In small open economies, policymakers are often concerned about external and internal sources of shocks and uncertainties, and whether their conduct of policy itself may contribute to economic fluctuations. In this paper, we ask how much do policy shocks contribute to Canadian business cycle fluctuations. Specifically, we ask: To what extent is this due to first-moment (i.i.d.) factors underlying structural shocks, and to what extent is this due to second-moment time-varying or stochastic volatility of these policy shocks? We address this policy-relevant question from the perspective of an estimated model. Using a structural business cycle model with stochastic volatility allows us to decompose and quantify which time-varying volatility of structural shocks matter. The internal logic of the model facilitates a clear identification and interpretation of structural shocks to domestic demand or supply conditions, as well as to domestic monetary or fiscal policy, and, international economic and policy spillovers factors. To our knowledge, this structural open-economy contribution to the "volatility shocks" literature is the first.

Our main findings are as follows. First, we identify considerable time-varying volatilities in domestic technology, markup, investment, fiscal-policy and monetary-policy shocks. The same holds for the exogenous block representing the foreign economy, in terms of foreign inflation, output-growth, and nominal interest rate shocks. There has been a noticeable increase in our identified domestic volatility shocks leading up to and after the Great Recession period (2008-2011). The variance of Canadian output can be largely attributed to domestic technology shocks (around 60 percent), and monetary policy shocks (around 20 percent) with international spillovers playing a smaller role (less than 1 percent).

When we further decompose the sources of these shocks to uncover the component due to the time-varying volatility in the shocks, we find that time-varying volatilities tend to dominate during times of turmoil. Our model-based shock-volatility accounting attributes: (i) the energy crisis of the early 1980s to international, cost-push, and monetary-policy shock volatilities; (ii) the adoption of inflation targeting in the early 1990s to monetary policy uncertainty; (iii) the recessions around the early 1980s and 1990s to investment volatility; and (iv) income-tax and capital-gains tax reform periods—in 1987 and 2000, respectively—to relatively large tax-policy uncertainty shocks in the model.

2 Related literature

The empirical literature on macroeconomic and policy volatility shocks go about measuring this notion in two ways. In the first method, one may use data constructs as proxies of time-varying volatilities in outcomes (see, e.g., Caggiano et al., 2014; Baker et al., 2016).[1] In the second, one may take a slightly more structural modelling approach where the statistical processes governing time-varying volatilities in economic and policy shocks have explicit economic interpretations (see, e.g., Bloom et al., 2007; Bloom, 2009; Born and Pfeifer, 2014; Fernández-Villaverde et al., 2015). In what follows, we take the second approach. By construction, we have well-defined (or identified) notions of domestic-versus-foreign economic and policy shocks. Additionally, this also allows us to avoid the well-known problems of weak identification of impulse dynamics in less theoretically-constrained statistical models (see, e.g., Yao et al., 2017, and the cited references therein). We use Canada as the case study.

To distinguish between domestic and foreign variations in economic and policy activity along with their associated uncertainty elements, we augment a typical incomplete-markets small open economy with NK features (e.g., Alonso-Carrera and Kam, 2016; Gali and Monacelli, 2005; Monacelli, 2005; Justiniano and Preston, 2010a), by allowing for time variation in the standard deviations of the structural shocks, á la Justiniano and Primiceri (2008). In terms of empirical methods, the paper closest to ours is Justiniano and Primiceri (2008). The authors consider a linearized medium-scale DSGE framework (see, e.g., Del Negro et al., 2007; Smets and Wouters, 2003; Christiano et al., 2005) and augment the structural shocks with time-varying volatility in the distributions of the shocks.[2] Using this framework to explore the potential causes of the Great Moderation, they conclude that reductions in the volatility of investment-specific technological shocks were a key driver of the reduction in real GDP volatility (see also, Fernández-Villaverde and Rubio-Ramírez, 2007; Bloom et al., 2007, for similar conclusions).[3]

More recently, research has moved beyond the role of the investment channel by investigating the role of domestic (monetary and fiscal) policy in shaping the business cycle. For instance Mumtaz and Zanetti

[1] Commonly used measures include the Chicago Board Options Exchange Market Volatility Index (i.e., the VIX) and the Economic Policy Uncertainty (EPU) index of Baker et al. (2016).

[2] We have also experimented with a higher-order, local nonlinear solution scheme to see if the time-varying volatilities interact with covariance and higher-order cross-effects. We find that the contribution of these terms are rather insignificant.

[3] In following Justiniano and Primiceri (2008), the stochastic-volatility components can be estimated along with the rest of the structural model using full-information Bayesian methods on a conditionally linear Gaussian state-space representation. However, to facilitate this computationally tractable method, one trades-off with accuracy of model solution and likelihood approximation. In our approach, as in Justiniano and Primiceri (2008), we solve the model to first-order accuracy by a standard perturbation method. We then approximate the originally non-linear and non-Gaussian conditional density of structural shocks by a mixture of Gaussian processes (Kim et al., 1998). Alternatively, one may prefer to trade-off estimation and computational speed in return for model accuracy: This can be done by solving the model using higher-order approximations of equilibrium policies, evaluating the non-Gaussian data likelihood of a resulting non-linear state-space representation and constructing the model's posterior density by sequential Monte Carlo. This latter method is costly, and as a result, researchers tend to use an incomplete-information approach to estimate the model (see, e.g., Fernández-Villaverde et al., 2011; Born and Pfeifer, 2014; Fernández-Villaverde et al., 2015): The authors would separately estimate the stochastic volatility (SV) processes, and then estimate the rest of the structural parameters of the DSGE conditioning on the estimated (SV) block. The second stage estimation is usually done using a partial-information method of simulated moments.

In our application, we think there is not much lost in terms of accuracy, since the decision problems faced by agents in our small open economy example is away—theoretically and in the observed Canadian data—from crucial sources of nonlinear dynamics like the zero lower bound on nominal interest. Hence, we conduct our analyses using the methods similar to Justiniano and Primiceri (2008).

(2013) find that monetary-policy volatility shocks have a negligible effect on real GDP (approximately 0.15 percent). A similar result which also encompasses the effects of fiscal-policy volatility shock is found by Born and Pfeifer (2014). Interestingly, when exploring a zero lower bound environment, Fernández-Villaverde et al. (2015) find that fiscal volatility shocks may decrease real GDP by around 1.5 percent. Our work here complements this literature by asking how important are time-varying volatilities in fiscal- and monetary-policy shocks for a small open economy, and how they vary over the recent history of a small open economy.

A related strand of literature in empirical macroeconometrics uses time-varying models to study the effect of shocks. This literature started with the development of time-varying structural vector autoregressions for the examination of monetary policy transmission in the US (Cogley and Sargent, 2001, 2005; Primiceri, 2005), but have been widely used in other applications. For instance, Benati and Surico (2009) shed light on the good policy vs good luck debate surrounding the Great Moderation, Mumtaz and Surico (2009) study the link between monetary policy, the yield curve and private sector behaviour and Mumtaz and Zanetti (2015) investigate the effects of technology shocks on the labor market.

Our research is also related to recent research on the effects of international spillovers. For example, Mumtaz and Theodoridis (2017) employ a dynamic factor model with stochastic volatility to show that cross-country volatility-shock spillovers have real effects among eleven OECD countries. Faccini et al. (2016) show that U.S. government spending has a significant spillover effect on its major trading partners. Our research complements both of these papers from the point of view of a small open economy. Beck and Jackson (2024) use a dynamic factor model to quantify the relative effects of international and regional factors in explaining international trade fluctuations in Canada.

Finally, our paper is also closely related to the literature on monetary policy evaluation using structural small open-economy models. For instance Lubik and Schorfheide (2007) and Kam et al. (2009) consider whether the monetary authorities of small open economies respond to variations in nominal exchange rates. A closely related analysis is conducted by Justiniano and Preston (2010b) who explore the optimal monetary policy design of the same three small open economies when the policy maker is faced with parameter uncertainty. In contrast to Justiniano and Preston (2010b), who study parameter uncertainty arising from estimation imprecision, we are interested in making inference about time-varying volatilities of structural shocks' distributions. Ravenna and Mølbak Ingholt (2021) use a DSGE model to determine whether inflation stabilization since the 1990s can be attributed to the adoption of inflation targeting or by a lucky period of relatively low volatility in business cycle shocks. They find that changes in expectations can explain the majority of inflation and output stabilization since the 1990s.

There is also a related literature on business cycle accounting. For example, Hevia (2014) studies real business cycle accounting for emerging and developed small open economies, in the spirit of Chari et al. (2007). In contrast, we consider a setting with explicit nominal frictions such that the model allows us to map structural first- and second-moments shocks to nominal and real outcomes. Our setting also allows us to identify monetary- and fiscal-policy variations.[4]

[4] Also, in Hevia (2014) and the real business cycle (RBC) accounting literature, one has a more agnostic measure of the "wedges" in equilibrium conditions in their RBC models. That is, the authors do not impose particular structural interpretations on these wedges. For example, the "labor wedge" may have a structural source through distortionary income tax policy or it may be because of market power on the side of firms. Here, we make stronger structural identification and interpretation of

3 Model

The small-open-economy model is a typical incomplete-markets New Open Economy Macroeconomics (NOEM) model (see Gali and Monacelli, 2005; Justiniano and Preston, 2010a; Alonso-Carrera and Kam, 2016). To identify a richer set of structural domestic market-side (investment, technology and cost-push) shocks, policy-side (monetary, labor-tax and capital-tax policy) shocks and foreign (output-growth, inflation and interest-rate) shocks, the model has a description of monetary policy, fiscal policy, and physical capital accumulation, in addition to standard NOEM elements. The structural shocks themselves are decomposable into first-moment components and second-moment or stochastic volatility components. This allow us to perform our volatility measurement and accounting from the lens of this model structure.

3.1 Representative household

The small open economy is populated by a continuum of identical households. Following McCallum and Nelson (1999) and Benigno and Thoenissen (2008), each household has access to a pair of non-state-contingent domestic and foreign money bonds, denoted B_t and B_t^*, which are respectively denominated in Home and Foreign currency. More precisely, if $\boldsymbol{s}_t$ denotes the aggregate state vector, then $B_{t+1}(\boldsymbol{s}_t)$ and $B_{t+1}^*(\boldsymbol{s}_t)$ respectively denote currency specific unit claims (e.g., one dollar) conditional on $\boldsymbol{s}_t$.[5] Thus, letting r_t and r_t^* respectively denote the domestic and foreign nominal interest rates, the date t cost of each bond in domestic currency terms are given by $(1+r_t)^{-1}$ and $S_t(\boldsymbol{s}_t)\left(1+r_t^*\right)^{-1}$, where $S_t(\boldsymbol{s}_t)$ is the nominal exchange rate expressed as domestic currency per unit of foreign currency. In what follows we reduce the notation on endogenous (random) variables by suppressing the arguments. More precisely, we define $\boldsymbol{X}_t := \boldsymbol{X}_t(s_t)$, where the function $\boldsymbol{s} \mapsto \boldsymbol{X}_t(\boldsymbol{s})$ is vector-valued, consisting of endogenous (i.e., equilibrium-determined) functions. For instance, $B_{t+1}^*(\boldsymbol{s}_t)$ is written more compactly as B_{t+1}^*.

Each household faces the sequential budget constraint:

$$P_tC_t + P_tI_t + \frac{B_{t+1}}{R_t} + \frac{S_tB_{t+1}^*}{(1+r_t^*)} \leq (1-\tau_{W,t})W_tN_t + (1-\tau_{K,t})R_{k,t}K_t + P_t\tau_{K,t}\xi K_t + B_t + S_tB_t^* + P_t\Theta_t, \quad (1)$$

where P_t is the domestic consumer price index, I_t is domestic investment, C_t is a CES composite index of home and foreign produced consumption goods later defined in (34), $R_t := (1+r_t)$ is the domestic (gross) nominal return on money holdings, $\tau_{W,t}$ is the marginal tax rate on labor income, W_t is the per hour nominal wage rate, N_t is the number of hours of labor supplied, $\tau_{K,t}$ is the marginal tax rate on capital income, K_t is the date t level of capital, $R_{k,t}$ is the per unit rental rate of capital, $\xi \in (0,1)$ is the capital depreciation rate and $\Theta \equiv \int_{[0,1]}\Theta(i)\mathrm{d}i$ is the total number of dividend payment received from ownership of all differentiated-product firms indexed by $i \in [0,1]$. The budget constraint (1) requires that the nominal value of consumption, investment, and new asset purchases, must be feasibly financed by post-tax capital and labor income, current holdings of Home and Foreign money claims, and profits from firm ownership.

the sources of the business cycle shocks.

[5] We will summarize what $\boldsymbol{s}_t$ comprises at the end of Section 4.

The law of motion for capital production is:

$$K_{t+1} = (1-\xi)K_t + \mu_t I_t \left[1 - \mathscr{D}\left(\frac{I_t}{I_{t-1}}\right)\right], \tag{2}$$

where μ_t is an investment shock which we discuss later, and, following Fernández-Villaverde et al. (2015), $\mathscr{D}(\cdot)$ is a convex cost function for capital adjustment of the form:

$$\mathscr{D}\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - \exp(g_A)\right)^2, \tag{3}$$

in which $\kappa \in (0,\infty)$ is the capital adjustment cost parameter, $\mathscr{D}(g_A) = \mathscr{D}'(g_A) = 0$, $\mathscr{D}''(g_A) = \kappa$, and, g_A denotes the total factor productivity growth factor along a balanced growth path.

Household preferences are represented by the total discounted expected utility criterion:

$$\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\delta_t U(C_t, N_t)\right\}, \qquad \delta_t := \begin{cases}\beta(C^a_{t-1}/A_{t-1})\delta_{t-1}, & \text{for } t>0\\ 1 & \text{for } t=0\end{cases} \tag{4}$$

where $\mathbb{E}_t := \mathbb{E}\{\cdot|\boldsymbol{s}_t\}$ is the linear expectations operator conditional on realized public information ($\boldsymbol{s}_t$) at the beginning of date t, A_t is realized total factor productivity (technology), C^a_t/A_t is (detrended) average consumption across households, and, δ_t is an endogenous discount factor.[6] We assume an additively separable utility function of the form: $U(C_t, N_t) := \frac{C_t^{1-\rho}}{1-\rho} + v(\tilde{G}_t) - \psi\left(A_t^{1-\rho}\right)\frac{N_t^{1+\varphi}}{1+\varphi}$, where $\rho > 0$ is the intertemporal elasticity of substitution, $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply, $\psi > 0$ is a scale parameter, $\tilde{G}_t$ is the (stationary) level of government spending and $v(\cdot)$ is an increasing, concave and bounded from above function. As in Fernández-Villaverde et al. (2015), the presence of technology in the utility function (i.e., A_t) ensures the existence of a balanced-growth path. Following Ferrero et al. (2010), the endogenous discount factor takes the following parametric form: $\beta(C^a_t/A_t) = \frac{\bar{\beta}}{1+\zeta[\ln(C^a_t/A_t)-\vartheta)]}$, where $\bar{\beta} \in (0,1)$. We will parametrize $\vartheta > 0$ and ζ such that that the endogenous discount factor has a negligible effect on the dynamics of the model, but they will matter enough to ensure the existence of a unique nonstochastic steady-state equilibrium.

The representative household chooses an optimal plan $\{C_t, N_t, B_t, B^*_t, K_{t+1}, I_t\}_{t\in\mathbb{N}}$ to maximize (4) subject to (1), taking the average level of consumption, nominal prices, policy rates and initial bonds holdings—i.e., $\{C^a_t, P_t, W_t, S_t, r_t, r^*_t\}_{t\in\mathbb{N}}, B_0, B^*_0$ and K_0—as given. The first-order conditions of this prob-

[6]The Uzawa (1968)-style endogenous discount factor function, $\beta : \mathbb{R}_+ \to (0,1)$, ensures that the model exhibits a unique nonstochastic steady state in the presence of incomplete markets and international borrowing and lending. For a survey on different approaches to introducing a deterministic steady state into small open economy models see Schmitt-Grohé and Uribe (2003).

lem at state $\boldsymbol{s}_t$ are characterized by the functionals:

$$A_t^{1-\rho}\psi N_t^{\varphi} C_t^{\rho} = (1-\tau_{W,t})\frac{W_t}{P_t}, \tag{5}$$

$$C_t^{-\rho} = R_t \mathbb{E}_t\left\{\beta\left(C_t^a/A_t\right)\left(\frac{P_t}{P_{t+1}}\right)C_{t+1}^{-\rho}\right\}, \tag{6}$$

$$C_t^{-\rho} = \left(1+r_t^*\right)\mathbb{E}_t\left\{\beta\left(C_t^a/A_t\right)\left(\frac{P_t^* Q_{t+1}}{P_{t+1}^* Q_t}\right)C_{t+1}^{-\rho}\right\}, \tag{7}$$

$$\lambda_t P_t = \tilde{q}_t\left(\left[1-\mathscr{D}\left(\frac{I_t}{I_{t-1}};g_A\right)\right]-\mu_t I_t \mathscr{D}'\left(\frac{I_t}{I_{t-1}};g_A\right)\right)-\delta_t \mathbb{E}_t\left\{\tilde{q}_{t+1}\mu_{t+1}I_{t+1}\mathscr{D}'\left(\frac{I_{t+1}}{I_t};g_A\right)\right\}, \tag{8}$$

$$\tilde{q}_t = \delta_t \mathbb{E}_t\left\{\lambda_{t+1}\left(\left(1-\tau_{K,t+1}\right)R_{K,t+1}+P_{t+1}\tau_{K,t+1}\xi\right)+\tilde{q}_{t+1}\left(1-\xi\right)\right\}, \tag{9}$$

where the Lagrange multiplier $\tilde{q}_t := \lambda_t q_{Nt}$ represents the nominal Tobin's-q: $q_{N,t}$, in terms of marginal utility: λ_t. In our Online Appendix, Section A, we define C_t as a CES composite of a continuous variety of home and foreign produced goods. This gives rise to a consumer price index:

$$P_t = \left[\left(1-\gamma\right)P_{H,t}^{1-\eta}+\gamma P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{10}$$

where $\eta > 0$ measures the elasticity of substitution between home and foreign goods indices.

3.2 Firm

As in Gali and Monacelli (2005), the production side of the economy consists of a continuum of retail firms $i \in [0,1]$, each of whom produce a differentiated product which is sold to the domestic government as well as domestic and foreign households, according to the demand schedule:

$$Y_{H,t+s}(i) = \left(\frac{P_{H,t+s}(i)}{P_{H,t+s}}\right)^{-\epsilon_{H,t}} Y_{H,t+s}, \qquad Y_{H,t+s} := C_{H,t+s}+I_{H,t}+G_{H,t+s}+C_{H,t+s}^*, \tag{11}$$

where $P_{H,t}$ is the domestic-goods producer price index, $Y_{H,t}$ is the aggregate level of domestic production, $C_{H,t}$, $I_{H,t}$ and $C_{H,t}^*$ respectively denote total levels of domestic and foreign consumption expenditure on home produced goods and expenditure, investment on domestic capital and $G_{H,t}$ is total level of government expenditure at any $t,s \in \mathbb{N}$. For simplicity, we assume that the Home government only consumes Home goods. The production technology is (constant returns to scale) Cobb-Douglas:

$$Y_{H,t}(i) = \left[A_t N_t(i)\right]^{1-\alpha}\left[K_t(i)\right]^{\alpha}, \tag{12}$$

where $\alpha \in (0,1)$, A_t is a labor-augmenting productivity term (later defined in (26)), $N_t(i)$ is the labor input and $K_t(i)$ is the capital input. Cost minimization with respect to capital and labor inputs implies that, in equilibrium, all intermediate good producing firms have the same capital-to-labor ratio and the same marginal cost of production:

$$MC_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{W_t^{1-\alpha}R_{K,t}^{\alpha}}{A_t^{1-\alpha}}, \tag{13}$$

$$\frac{K_t(i)}{N_t(i)} = \left(\frac{\alpha}{1-\alpha}\right)\frac{W_t}{R_{K,t}}, \tag{14}$$

where MC_t is the nominal marginal cost (i.e., the shadow value of, or Lagrange multiplier on, the firm's technology constraint).

Since firms compete in monopolistically competitive environment, they must also decide the price to charge for their variety of good. Following Rotemberg (1982), we assume that each firm faces a convex price-adjustment cost:

$$AC\left(\frac{P_{H,t+s}(i)}{P_{H,t+s-1}(i)}, Y_{H,t+s}(i)\right) := \frac{\varpi}{2}\left(\frac{P_{H,t+s}(i)}{P_{H,t+s-\Pi}(i)} - \Pi\right)^2 \times Y_{H,t+s}(i),$$

where Π is gross CPI inflation along a deterministic balanced-growth path (i.e., the monetary authority's inflation target). The parameter ϖ controls the degree of price stickiness. If $\varpi = 0$ then prices are fully flexible. Thus, a larger ϖ, implies more stickiness in pricing. Since this part of the model is standard, we relegate the firm's pricing decision problem to Section A of our Online Appendix.

3.3 Market clearing

There are four types of Walrasian markets in our environment: A continuum of domestic labor and capital markets, a continuum of internationally traded goods market, and the international asset markets trading in non-state-contingent money claims. We consider each in turn.

First, under the assumption that labor and capital are immobile across countries, the domestic labor and capital markets must clear in a competitive equilibrium. Equating labor supply (5) and demand (13), (14) gives:

$$A_t^{1-\rho}\psi N_t^{\varphi} C_t^{\rho} = (1-\tau_{W,t})mc_{H,t}A_t p_{H,t} \tag{15}$$

where we have defined $p_{H,t} = \frac{P_{H,t}}{P_t}$ and $mc_{H,t} = \frac{MC_t}{P_{H,t}}$. Similarly, equating capital supply (8), (9) and demand (13), (14) gives the capital market clearing condition.

Second, goods market clearing for each variety of good $i \in [0,1]$, accounting for the resource cost of price adjustments, yields the condition:

$$\left[1 - \frac{\varpi}{2}\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)} - \Pi\right)^2\right] Y_{H,t}(i) = C_{H,t}(i) + I_{H,t}(i) + G_{H,t}(i) + C^*_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_{H,t}} Y_{H,t} \tag{16}$$

where $Y_{H,t} = \left[(1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}(C_t + I_t + G_t) + \gamma\left(\frac{P_{H,t}(i)}{S_t P^*_{F,t}}\right)^{-\epsilon_{F,t}}\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t\right]$, is the sum of Marshallian demands for good i, by Home households and government, and also by Foreigners. The third equality is derived from the demands for Home index goods by the same agents, which embed the assumption that Foreign and Home agents have symmetric preference representations.

Let us define an aggregate Home output index: $Y_t = \left[\int_0^1 Y_{H,t}^{(\epsilon_{H,t}-1)/\epsilon_{H,t}}(i)\mathrm{d}i\right]^{\epsilon_{H,t}/(\epsilon_{H,t}-1)}$. The aggregate goods market clearing condition is

$$\left[1-\left(\Pi_{H,t}-\Pi\right)^2\right]Y_t = Y_{H,t} \equiv \left(p_{H,t}\right)^{-\eta}\left[(1-\gamma)(C_t+G_t)+\gamma Q_t^{\eta}C_t^*\right]. \tag{17}$$

3.4 Government behavior and policy shock processes

We close the model by describing monetary and fiscal policies as following simple policy rules. The monetary authority follows a conventional Taylor-type rule:

$$\frac{R_t}{R} = \frac{R_{t-1}}{R}^{\phi_R}\frac{\Pi_t}{\Pi}^{(1-\phi_R)\phi_\Pi}\frac{Y_t}{YA_t}^{(1-\phi_R)\phi_Y}\exp\left\{\sigma_{R,t}\varepsilon_{R,t}\right\}, \qquad \varepsilon_{R,t}\sim\mathcal{N}(0,1), \tag{18}$$

where $\phi_R \in [0,1)$ models the degree of interest-rate smoothing behavior, while $\phi_\Pi > 0$ and $\phi_Y \geq 0$ respectfully model the monetary authority's response to CPI inflation ($\Pi_t := P_t/P_{t-1}$) and contemporaneous output Y_t (to be defined later). Terms in the denominators without a subscript denote steady-state levels or rates of their respective variables in the numerator. As in Primiceri (2005), the structural shock $\varepsilon_{R,t}$ captures "non-systematic monetary policy", interpretable as "policy mistakes", as well as any policy actions that are left unexplained by the model. The time varying volatility term $\sigma_{R,t}$, thus allows for time-varying second moments in the distribution of non-systematic variations in monetary policy. For instance, institutional changes such as the change from monetary to inflation targeting may result in a lower value for $\sigma_{R,t}$.

For simplicity, we follow Fernández-Villaverde et al. (2015) and assume that the fiscal authority does not accumulate a stock of debt, so the government budget constraint is always balanced:

$$G_t = \tau_{W,t}\frac{W_tN_t}{P_{H,t}} + \tau_{K,t}\frac{R_{K,t}K_t}{P_{H,t}}. \tag{19}$$

Following Fernández-Villaverde et al. (2015), the (capital and labor) income tax rates are modeled as a mean-reverting tax smoothing rule:

$$\tau_{i,t}-\tau_i = \alpha_i\left(\tau_{i,t-1}-\tau_i\right)+\phi_{i,Y}\left(\frac{Y_t}{Y_{t-1}}-1\right)+\exp\left\{\sigma_{\tau_i,t}\right\}\varepsilon_{\tau_i,t} \qquad \varepsilon_{\tau_i,t}\sim\mathcal{N}(0,1), \tag{20}$$

in which $i \in \{K, W\}$, τ_i is the steady state tax rate, $\alpha_i \in [0,1)$ is a stationary autoregressive coefficient and $\phi_{i,Y} > 0$ is a feedback effect from the current state of the business cycle. As was the case in the Taylor rule, $\varepsilon_{\tau_i,t}$ captures unanticipated changes to fiscal policy. This component can be thought of as exogenous political changes arising in the implementation of fiscal policy. The term $\sigma_{\tau_i,t}$, specified in (31), captures time-varying volatility in these non-systematic fiscal policy shocks.

3.5 Competitive equilibrium

Asset pricing. The requirement of zero profitable arbitrage in equilibrium is given by the equality between the Euler functionals (6) and (7). This can be rewritten as:

$$R_t \mathbb{E}_t \left\{ \beta \left(C_t^a / A_t \right) \left(\frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\rho} \right\} = \mathbb{E}_t \left\{ \beta \left(C_t^a / A_t \right) \left(\frac{Q_{t+1}}{Q_t} \right) C_{t+1}^{-\rho} R_{t+1}^* \right\}, \tag{21}$$

which implies uncovered interest parity (UIP) condition. Since the process for $R_t^* := 1 + r_t^*$ is (exogenously) given as an AR(1)-SV model, this asset pricing condition will exhibit an exogenous time-varying risk component.

Phillips curve. We restrict attention to a symmetric equilibrium: All firms $i \in [0,1]$ will choose a pricing strategy such that at each date t and state $\boldsymbol{s}_t$, $P_{H,t}(i) = P_{H,t}$. Denote $\Pi_{H,t} := P_{H,t}/P_{H,t-1}$. After some algebra, the firms' optimal pricing condition (shown in our Online Appendix, Section A) implies an equilibrium "Phillips curve" functional equation:

$$\begin{aligned} &\Pi_{H,t}(\Pi_{H,t} - \Pi) - \frac{\epsilon_{H,t}}{2}(\Pi_{H,t} - \Pi)^2 = \\ &\qquad \beta (C_t^a / A_t) \mathbb{E}_t \left\{ \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}} \left(\Pi_{H,t+1} - \Pi \right) \Pi_{H,t+1} \cdot \frac{Y_{H,t+1}}{Y_{H,t}} \right\} + \frac{\epsilon_{H,t}}{\varpi} \left[mc_{H,t} - \frac{\epsilon_{H,t} - 1}{\epsilon_{H,t}} \right]. \end{aligned} \tag{22}$$

This is an expectations-augmented Phillips curve. Also, the greater is the cost of prices adjustment, $\varpi \to \infty$, the gap between expected discounted next-period marginal (profit) value of inflation and current marginal value of inflation goes to zero. That is, prices are expected not to change very much (i.e., are not sensitive to real marginal cost deviations) the more costly is price adjustment. The greater is the elasticity of demand, $\epsilon_{H,t} \to +\infty$, the more positive and sensitive is the response of current inflation to real marginal cost (limiting case of perfect competition) deviation. Note that $(\epsilon_{H,t} - 1)/\epsilon_{H,t}$ is the inverse of a monopolist's static optimal markup, which depends on the firm's demand elasticity $\epsilon_{H,t}$.

Useful identities. From the CPI index (10), we can derive the Home final goods price index relative to the CPI index as:

$$p_{H,t} := \frac{P_{H,t}}{P_t} = \left[\frac{1 - \gamma (Q_t)^{1-\eta}}{1 - \gamma} \right]^{\frac{1}{1-\eta}}, \tag{23}$$

where we have used the definitions $P_{F,t}/P_t = S_t P_t^* / P_t =: Q_t$. As a corollary, we have that $p_{H,t}/p_{H,t-1} = (P_{H,t}/P_{H,t-1})/(P_t/P_{t-1})$, which implies:

$$\Pi_t = \frac{\Pi_{H,t}}{p_{H,t}/p_{H,t-1}} = \Pi_{H,t} \times \left[\frac{1 - \gamma (Q_{t-1})^{1-\eta}}{1 - \gamma (Q_t)^{1-\eta}} \right]^{\frac{1}{1-\eta}}. \tag{24}$$

Aggregating (12) up, we have

$$Y_{H,t} = [A_t N_t]^{1-\alpha} [K_t]^{\alpha}. \tag{25}$$

Given these identities, a recursive competitive equilibrium is defined as follows:

Definition 1. Given policies (18) and (20), a *recursive competitive equilibrium* is a system of allocation functions $\boldsymbol{s}_t \mapsto (C, I, N, K, G, Y_H, mc_H)(\boldsymbol{s}_t)$, and pricing functions $\boldsymbol{s}_t \mapsto (\Pi_H, p_H, \Pi, Q)(\boldsymbol{s}_t)$, such that:

1. Households optimize: (5), (6), (7), (8), (9), (23) and (24);

2. Firms optimize: (13), (14), (22) and (25);

3. Markets clear (given agents optimize): (8), (9), (13), (14), (15), and (17);

4. Government budget constraint holds: (19);

$C_t^a = C_t$, and $\lim_{t\to\infty} \delta_t R_t \mathbb{E}_t \left\{C_{t+1}^{-\rho} \Pi_{t+1}^{-1} B_{t+1}\right\} = \lim_{t\to\infty} \delta_t \mathbb{E}_t \left\{C_{t+1}^{-\rho} Q_{t+1} R_{t+1}^* B_{t+1}^*\right\} = 0$, for each date $t \in \mathbb{N}$ and state $\boldsymbol{s}_t$.

Since the labor augmenting technology process A_t has a unit root, consumption, labor, government expenditure and output all evolve along the stochastic growth path. Thus, before solving the model, we first need to solve for the competitive equilibrium in terms of stationary allocation and pricing functions. To do so, we define stationary functions by taking the ratio $\tilde{X}_t = X_t / A_t$, where $X \in \{C, N, G, Y\}$. The characterization of Definition 1 in stationary terms is provided in the online appendix.

4 Exogenous Stochastic Processes

Domestic monetary- and fiscal-policy shocks. We have already alluded to two sources of exogenous structural shocks acting through domestic monetary and fiscal-policy, respectively, in (18) and (20). Their statistical models will be given below. Before getting there, we will complete the description of the rest of the exogenous shock processes that shift the model economy.

Domestic technology, cost-push and investment shocks. The law of motion for labor-augmenting technology (A_t), cost-push (ϵ_H) and investment (μ) shocks is a mean reverting process:

$$\ln(i_t) = \left(1 - \rho_i\right) i + \rho_i \ln(i_t) + \sigma_{i,t} \varepsilon_{i,t}, \qquad \varepsilon_{i,t} \sim \mathcal{N}(0,1), \tag{26}$$

where $\rho_i \in (0,1)$ is an AR(1) coefficient, $i \in \{g_A, \epsilon_H, \mu\}$, $g_{A,t} := \frac{A_t}{A_{t-1}}$ is the gross growth rate of technology and g_A and ϵ_H denote, respectively, the rate of technological growth and mark-up along the balanced growth path.[7] The fact that (26) has time varying volatility means that the riskiness of the future paths of domestic technology growth, mark-ups and investment is permitted to change over the course of the business cycle.

[7] The unconditional mean of the markup shock μ is set to zero.

The rest of the world. We assume that the rest of the world can be modeled as the limit of a large closed economy. Thus, $C_t^* = Y_t^*$ is the rest of the world's output. The rest of the world is assumed to follow a recursively-ordered, second-order VAR-SV process:[8]

$$\mathbf{L}\mathbf{Z}_t^* = \beta_1 \mathbf{Z}_{t-1}^* + \beta_2 \mathbf{Z}_{t-2}^* + \mathbf{w}_t, \qquad \mathbf{w}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{F}_t) \tag{27}$$

where $\mathbf{Z}_t^* := [\Delta \ln(Y_t^*), \pi_t^*, r_t^*]'$ lists the (demeaned) percentage growth in foreign real GDP, foreign inflation rate, and, foreign nominal interest rate. The β_i objects, for $i = 1, 2$, are 3×3 matrices of VAR coefficients. Also,

$$\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 \\ \sigma_{\pi^*, Y^*} & 1 & 0 \\ \sigma_{r^*, Y^*} & \sigma_{r^*, \pi^*} & 1 \end{bmatrix}, \text{ and, } \mathbf{F}_t = \begin{bmatrix} \sigma^2_{Y^*, t} & 0 & 0 \\ 0 & \sigma^2_{\pi^*, t} & 0 \\ 0 & 0 & \sigma^2_{r^*, t} \end{bmatrix}. \tag{28}$$

As is standard in the VAR literature, interest-rate (i.e., monetary policy) shocks are assumed to be independent of any other innovations, however the ordering of the non-policy block is somewhat arbitrary (see, e.g., Primiceri, 2005). For robustness we therefore ensured that the results are not subject to ordering effects.

4.1 Structural shocks and volatility shocks

Let $\tilde{\boldsymbol{u}}_t$ denote an 9×1 vector collecting all the policy and economic disturbances—i.e., the structural shocks:

$$\tilde{\boldsymbol{u}}_t = \boldsymbol{\Sigma}_t^{1/2} \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_9), \tag{29}$$

where $\mathbf{I}_9$ is a (9×9) identity matrix,

$$\boldsymbol{\Sigma}_t = \begin{bmatrix} \mathbf{F}_t & \mathbf{0}_{(3\times 6)} \\ \mathbf{0}_{(6\times 3)} & \mathbf{D}_t \end{bmatrix}, \text{ and, } \mathbf{D}_t = \begin{bmatrix} \sigma^2_{A,t} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{R,t} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\tau_W,t} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{\tau_K,t} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{\epsilon_H,t} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{\mu,t} \end{bmatrix}. \tag{30}$$

Note that when we abuse notation and write $\boldsymbol{\Sigma}_t^{1/2}$, it is understood that $\boldsymbol{\Sigma}_t$ is a diagonal matrix. Each element of the stochastic volatilities, $\boldsymbol{\Sigma}_t$, evolves according to the stochastic process:

$$\log \sigma_{i,t} = (1 - \psi_i)\varphi_{i,t} + \psi_i \log \sigma_{i,t-1} + \nu_{i,t}, \qquad \nu_{i,t} \sim \mathcal{N}(0, \omega_i^2), \tag{31}$$

[8] In our model estimation, we estimate this process. We arrive at this choice of lag length for the foreign VAR-SV model using the Bayesian information criterion (BIC).

where $\varphi_{i,t}$ is the conditional mean, and, $|\psi_i| < 1$ for $i \in I := \{A, R, \tau_W, \tau_K, \epsilon_H, \mu, Y^*, \pi^*, r^*\}$.[9] Denote the collection of variance parameters as $\boldsymbol{\omega} = \{\omega_i\}_{i \in I}$.

Discussion. The vector of states relevant to agent decisions is $\boldsymbol{s}_t = \left(B_t, B_t^*, K_t, A_t, \mathbf{Z}_t^*\right)$. Also, from (29) and (31), we can see that there are two sources of variations in structural shocks, $\tilde{\boldsymbol{u}}_t$. For each shock i, one source of innovation, $\varepsilon_{i,t} \in \boldsymbol{\varepsilon}_t$, is a Gaussian shock to the policy or economic variable itself—a *mean* structural shock. Another component of innovation, $\nu_{i,t}$, is *volatility shock*, which renders a permanent shock to the spread of the distribution of each $\tilde{u}_{i,t} \in \tilde{\boldsymbol{u}}_t$.

5 Model Solution and Empirical Observables

From a stationarized version of Definition 1, its implied deterministic steady-state equilibrium is computed by the steps listed in the online appendix. Following Justiniano and Primiceri (2008), the model's recursive competitive equilibrium conditions are approximated by a perturbation method which is accurate to first-order.[10] We then use a standard rational expectations equilibrium (REE) algorithm to find the stable REE solution, represented as a conditionally linear and Gaussian state-space system, and map observed data to it as

$$\begin{aligned}\mathbf{x}_{t+1} &= \mathbf{A}_\theta \mathbf{x}_t + \mathbf{B}_\theta \tilde{\boldsymbol{u}}_t,\\ \mathbf{y}_t^o &= \mathbf{H}^o \mathbf{y}_t,\end{aligned} \tag{32}$$

where $\mathbf{x}_t$ is a vector of all endogenous variables in the system, $\mathbf{y}_t^o$ is a vector of observables, and, $\mathbf{H}^o$ is the linear observation equation.[11]

Let $\Delta \log X_t$ denote the first difference $\log X_t - \log X_{t-1}$. For the observables,

$$\mathbf{y}_t^o := (\Delta \log Y_t + g_{A,t}, \log R_t, \log \tau_{W,t}, \log \tau_{K,t}, \log \Pi_t, \Delta \log Q_t, \Delta \log Y_t^*, \pi_t^*, r_t^*),$$

Y_t denotes the level of real GDP per capita in the SOE, R_t is the domestic gross nominal interest rate, $\tau_{W,t}$ is the marginal labor tax rate, $\tau_{K,t}$ is the marginal capital tax rate, Π_t is the gross inflation rate, Q_t is the real exchange rate in terms of CAD per USD, Y_t^* is the international level of real GDP per capita, π_t^* is

[9] The reason for modeling the stochastic processes (31) in logarithms, is to ensure that the random levels of the standard deviations, $\sigma_{i,t}$, remain positive almost everywhere. This law of motion for the conditional volatilities ensures the existence of closed form solutions for the first and second moments (Andreasen, 2010), and has been used in, e.g., Born and Pfeifer (2014).

[10] One might conjecture that the SV effects should also interact directly with the model's endogenous dynamics (e.g., through time-varying covariance and risk-premium terms). To address this, we have experimented with alternative higher (third) order perturbation solutions. However, the resulting higher-order dynamics (evaluated at our mean posterior estimates) seem to be negligible. Therefore, we focus on a linearized solution with SV shocks in the rest of the paper. Our chosen solution strategy also means that we can carry out full-information estimation feasibly. A limitation of a nonlinear solution means that performing full-information estimation will be very costly, since evaluating the model's data likelihood function will require very costly particle filtering. Existing papers featuring nonlinear solutions rely on incomplete-information estimators such as those from the method of simulated moments.

[11] Such state-space representations of the dynamical system are typically used to estimate DSGE models using either maximum likelihood (see, e.g., Ireland, 2004; Zanetti, 2008) or Bayesian methods (see, e.g., An and Schorfheide, 2007; Herbst and Schorfheide, 2015).

the foreign inflation rate, and, r_t^* is the foreign nominal interest rate. Recall that domestic output in our model is defined as the ratio of output to productivity A_t. Hence, observed output growth corresponds to $\Delta \log Y_t$ adjusted by productivity growth $g_{A,t}$. All data for these observable variables are quarterly. The sample period under investigation is from 1981 (Quarter 1) to 2018 (Quarter 4). Data sources and transformations of each series used in the empirical analysis are provided in the online appendix.

In summary, the implied econometric model is jointly given by equations (26), (27), (28), (29), (30), (31), and (32).

6 Bayesian Estimation

We now outline a method for evaluating the non-analytical posterior joint distribution of the implied econometric model given observed data.[12] To obtain posterior draws for the model's structural parameters $(\boldsymbol{\theta}, \boldsymbol{\omega}^2)$ and the time varying volatilities $(\{\boldsymbol{\Sigma}_t\}_{t=1}^T)$, our estimation procedure uses a four-step Metropolis-within-Gibbs algorithm. In the first step, given $\boldsymbol{\omega}^2$ and $\{\boldsymbol{\Sigma}_t\}_{t=1}^T$, the model's microeconomic parameters $\boldsymbol{\theta}$ are drawn (updated) through a random walk Metropolis-Hastings algorithm as in Schorfheide (2000). Next, the structural shocks $\{\tilde{\boldsymbol{u}}\}_{t=1}^T$ are simulated using the efficient disturbance smoother developed by Durbin and Koopman (2002). Drawing the time-varying volatilities requires the combination of two procedures: In the first step, we apply the auxiliary mixture sampler of Kim et al. (1998) to approximate the underlying non-linear, non-Gaussian state-space representation as a mixture of linear Gaussian models.[13] Following this, the volatilities $\boldsymbol{\Sigma}_t$ can then be sampled with standard linear Gaussian methods as in Carter and Kohn (1994). However, here we make use of an efficient algorithm by Chan and Hsiao (2014) which takes advantage of the fact that the precision matrices of the underlying state space model are both block-banded and sparse. Conditional on the above blocks, the posterior distributions of the remaining parameters, $(\boldsymbol{\omega}^2)$, have analytical Inverse Gamma density representations. This Gibbs-sampling with conditional blocking method is known to induce the correct posterior density of the model's parameters $(\boldsymbol{\theta}, \boldsymbol{\omega}^2)$ (see Stroud et al., 2003; Del Negro and Primiceri, 2015).

6.1 Our priors on the model

The prior distributions of each of the structural parameter in the DSGE-SV model are provided in Table 1.

[Table 1 about here.]

The associated posterior estimates will be discussed in the next section. Here we note that all prior distributions are assumed to be independent and the support of each parameter's density is restricted to

[12] This is similar to the method used in Justiniano and Primiceri (2008), with the exception of the penultimate step where we utilize a more efficient smoother to construct a sequence of stochastic volatilities conditional on other estimated blocks in the Gibbs sampler. More details are presented in Appendix D in the online appendix.

[13] The stochastic-volatility components of the model renders nonlinearity and non-Gaussian (χ^2) distributions in its state-space representation, which can be approximated as a log-linear Gaussian mixture process.

be in line with economic theory. For instance, the autoregressive term in the labor augmenting technology equation follows a beta distribution on the unit interval. When choosing the values for the hyperparameters we lean on existing studies were possible. The prior distributions of the domestic economies DSGE parameters follow from Justiniano and Preston (2010a) who also estimate a small open economy model using data from Canada and the US, but without stochastic volatility. The hyperparameters for the parameters in the state equations associated with the stochastic volatilities are standard within the broader literature on empirical stochastic volatility models (see, e.g., Chan and Hsiao, 2014). Similarly, following common practice in the VAR literature, priors for the VAR coefficients are made uninformative by choosing hyperparameters such that they are centered upon zero with a large variance (see, e.g., Chan and Eisenstat, 2017).

Given that we are estimating a large model, certain parameters are not estimated but instead fixed to conventional values. This includes setting $\epsilon_H = 8$, which implies a steady state markup of 14 percent.[14] The steady state discount factor $\bar{\beta} = 0.99$ and the parameters in the endogenous discount factor function are set to $\varphi = 1.27$ and $\vartheta = 10^{-6}$. In practice we found that g_A was not well identified by the data. Following Fernández-Villaverde et al. (2015), we therefore set $g_A = 0.005$ which implies a steady state growth rate of technology to 2 percent per annum.

7 Estimation Results

We will address our main question on the accounting of macroeconomic and policy volatility shocks on a small open economy's real GDP growth in Section 7.3 below. Readers interested in the economic conclusions may go directly there from here. Otherwise, we will discuss the model estimation results here.

7.1 Structural parameter estimates

The posterior mean and standard-deviation statistics of of the estimated parameters are reported alongside their corresponding prior densities' statistics in Table 1. We have tested to ensure that the posterior distributions represent an ergodic distribution of the Markov chain (induced by our Metropolis-within-Gibbs sampler). In the interest of brevity, these convergence diagnostics are summarized in our online appendix. Here we note that all parameters are quite precisely estimated as evidenced by the small standard deviations. However, some structural parameters have posteriors that do not vary much from their priors. This may suggest that some model micro-parameters may not be well-identified. We have experimented with fixing some of these parameter values and found that this does not change our overall results. As such, we are not too concerned about some of the posteriors being numerically close to their priors, which are in line with those from Justiniano and Preston (2010a).

Interestingly, despite having a more general modeling framework, our estimated posterior means for the Taylor-rule parameters are close to those in Justiniano and Preston (2010a). Regarding our description of the fiscal policy rules, the estimates show that the smoothing parameters are quite similar. In

[14] To see this, note that the percentage markup in steady state markup is given by $\frac{\epsilon_H}{\epsilon_H - 1} - 1$.

contrast, the labor tax is weakly correlated with output fluctuations, the capital tax is almost invariant. While the latter result is consistent with existing studies on the US economy (Born and Pfeifer, 2014; Fernández-Villaverde et al., 2015), the former result suggests that there are differences between in the US and Canadian fiscal policy responses.

7.2 Behavior of shock uncertainties

Now we examine the economic significance of allowing for time varying volatilities in each of the shocks. Figure 1 displays the posterior mean of the estimated stochastic volatilities (blue solid graphs), over the sample period. The 16-th and 84-th percentiles of the respective posteriors are depicted as dashed-red graphs. The initial decline in volatility across most variables following 1980 is consistent with both DSGE

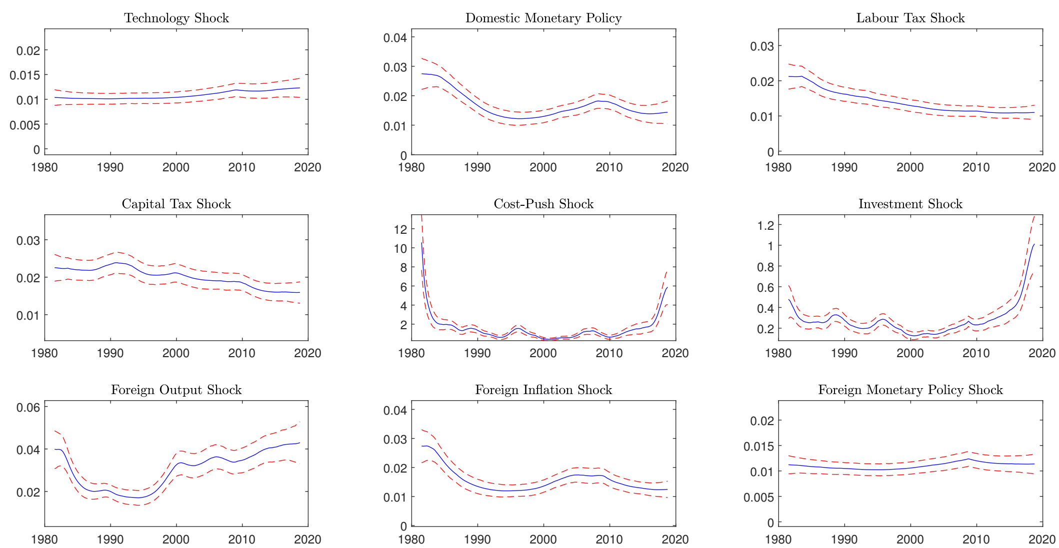

Figure 1: Estimated stochastic volatility $(\sigma_{i,t})$, where $i \in \{A, R, \tau_W, \tau_K, \epsilon_H, \mu, Y^*, \pi^*, r^*\}$. We deem the structural shocks to technology, cost-push, and investment as (domestic) "market-side shocks" and shocks to monetary, labor-tax and capital-tax policies as "policy-side shocks."

and VAR evidence on the Great moderation period (Primiceri, 2005; Justiniano and Primiceri, 2008).[15] With respect to the domestic economy, technology and the two tax rates are relatively stable, with the latter exhibiting a slight downward trend across the sample period. The domestic monetary policy rate exhibits an initial reduction in volatility associated with the adoption of inflation targeting in 1991, and then increased around the 2000 recession and remained high until after the Great Recession. Finally, the bulk of domestic volatility stems from cost-push and investment shocks. This is consistent with closed-economy results for the US economy (Justiniano and Primiceri, 2008).

The results for the US shocks differ from those found in VAR studies (Primiceri, 2005; Mumtaz and

[15] We highlight the fact that these results are robust to model variants where we estimate the initial condition: $\sigma_{i,0}$, in (31) or draw it from a known distribution with large variance, i.e. $\sigma_{i,0} \sim N(0, V_{\sigma_0})$ where $V_{\sigma_0} = 10$.

Zanetti, 2013). Aside from the difference in sample size, one possible explanation for these differences is the fact that the aforementioned VAR studies utilize time-varying-coefficient VAR-SV models, compared to our DSGE model which implies a constant-coefficient VAR-SV reduced form. For instance, the US monetary policy shock volatility presented here is qualitatively similar that presented in Born and Pfeifer (2014) - which was derived from a first-order autoregressive AR(1) model with SV. We also note that the choice of a VAR-SV for the (exogenous) US economy is in line with recent results in Chan and Eisenstat (2017).[16]

7.3 Volatility shock accounting

We address the main question in the paper here by decomposing and quantifying which time-varying volatility—in domestic demand or supply conditions, in domestic monetary or fiscal policy, or, in international economic and policy spillovers factors—matter for a small open economy.

To this end, we analyze the variance decomposition of domestic real GDP. Following Justiniano and Primiceri (2008), this is conducted as follows: First, we use the state space representation of the model solution to construct the implied variances of the endogenous observable variables conditional on a draw of the parameters and volatilities. Second, we construct time-varying variance decompositions by sequentially setting to zero the volatility of all disturbances but one, for all time periods. Since the objective of our analysis is to decompose the effects of volatility shocks and i.i.d. shocks on real GDP, we do not present the graphs for the remaining variance decompositions.

Total variation. Figure 2 presents the evolution of the (long-run) forecast error variance shares of GDP growth attributed to each of the structural shocks in (32). The blue (solid) line represents the average share and the red (dotted) lines are associated 16-th and 84-th percentiles.[17] In line with existing empirical studies, we find that technology shocks are the dominant driver of Canadian real GDP fluctuations (about 60 percent on average) (see, e.g., Watanabe, 2012). In contrast with existing studies on the US economy (see, e.g., Mumtaz and Zanetti, 2013; Born and Pfeifer, 2014; Fernández-Villaverde et al., 2015), domestic policy shocks contribute a non-negligible share of Canadian output volatility.

The largest contribution in terms of policy shocks stems from domestic monetary policy shocks (about 20 percent on average), with capital and labor tax shocks respectively accounting for approximately equivalent shares (about 5 percent each on average). Interestingly, despite having the largest volatilities in terms of magnitude, the cost-push shocks account for a negligible proportion of domestic real output fluctuations (around 2 percent of average), while the investment shock accounts for majority of the remaining variation (around 6 percent on average). The reason for this is a weak propagation mechanism in the cost-push shocks. Consistent with Justiniano and Preston (2010a) we also find that the identified international spillovers are substantially less than those found in SVAR studies (e.g., Cushman and Zha, 1997). Another symptom of this appears in the model not being able to fully rationalize observed

[16] Using the same observable time series for the US economy as those in this paper, Chan and Eisenstat (2017) show that the VAR-SV model has superior in-sample fit compared to its various model counterparts—e.g., a VAR-SV with time-varying-coefficients and a constant VAR.

[17] In Table 3 of our Online Appendix G, we provide additional descriptive statistics for the estimated distributions of the relative shares of stochastic-volatility in structural shocks over the sample period.

empirical cross-correlations between domestic and foreign variables. While the estimated model can rationalize somewhat the direction of correlation between the real exchange rate and the U.S. variables, it tends to miss the contemporaneous correlation between Canadian Output and U.S. data.[18] To conserve space, we defer further details on this result to Appendix F in the online appendix. Finally, we end this section by noting that the large positive correlation observed between the time varying volatility of the shocks, and their associated contribution to the unconditional variance of real GDP, is in line with results in Justiniano and Primiceri (2008).

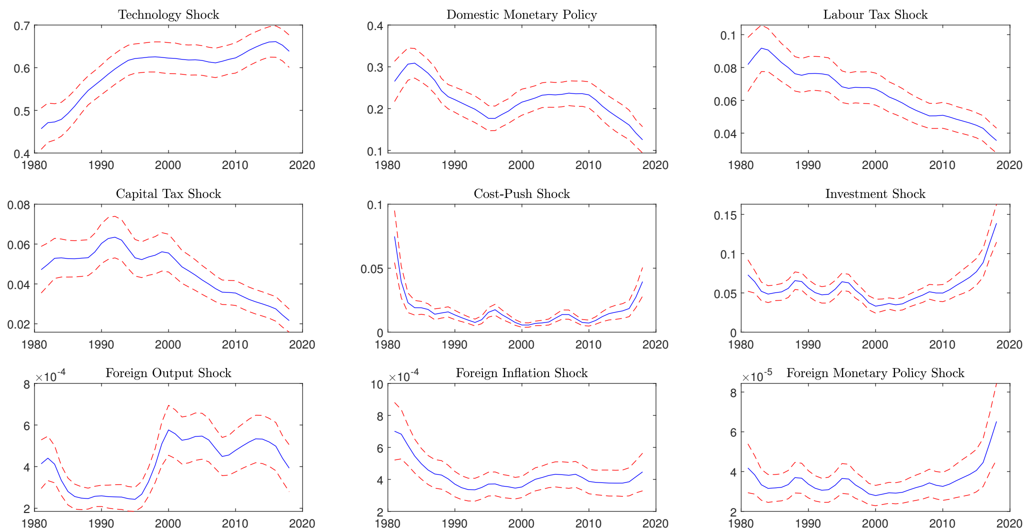

Figure 2: (Long-run) Variance Decomposition for Real GDP Growth (40 quarters ahead). We deem the structural shocks to technology, cost-push, and investment as (domestic) "market-side shocks" and shocks to monetary, labor-tax and capital-tax policies as "policy-side shocks."

Instead of just looking at the long-run variance decomposition (of real GDP growth) we can also break this up into short-to-medium-run horizons (h) for the variance decompositions. In Figure 3, we show sample slices of the decomposition at particular horizons $h \in \{1, 4, 8, 20, 40\}$ following each sampled time-varying shock in a particular quarter (measured on the horizontal axis). As we step forward in the forecast horizon h, we see the decomposition profiles or graphs converging onto the long run graphs shown in Figure 2. Figure 3 basically confirms the same insight as the long-run case of Figure 2, for the cases of short- and medium-term forecast horizons.[19] A key additional insight is that in the short-run, monetary

[18] One reason that we abstract the Foreign block of the model (into a simple structure featuring just output, inflation and monetary policy from the U.S.) is that there is existing empirical evidence to that effect. A recent structural VAR study for Canada by Ha and So (2023) also suggests quite robust evidence that U.S. monetary-policy have small effects on Canada. The authors show quite robustly that much of the foreign variations get absorbed by counteracting movements in the trade balance and the exchange rate. Another reason is for modelling and estimation tractability. Our strategy here follows existing literature (see, e.g., Justiniano and Preston, 2010a; Fernández-Villaverde et al., 2011; Hevia, 2014). In earlier versions of our work, we have also considered more foreign factors and multiple foreign-country variables. However, the benefit of having more detailed models was quickly overwhelmed by the cost of over-parametrization, with symptoms of poorly-identified and imprecisely-estimated structural parameters.

[19] Alternatively, Table 4 in our Online Appendix G provides a summary of the information behind Figure 3 in terms of averaging

policy shocks have a greater impact on the real economy. However, in the long-run technology shocks then take over as the most important factor. This is in line with the notion of the former shocks having a transitory effect on the economy, while the latter have a longer lasting permanent impact. For similar reasons, we also see the effects of the tax shocks decreasing in the horizon (h).

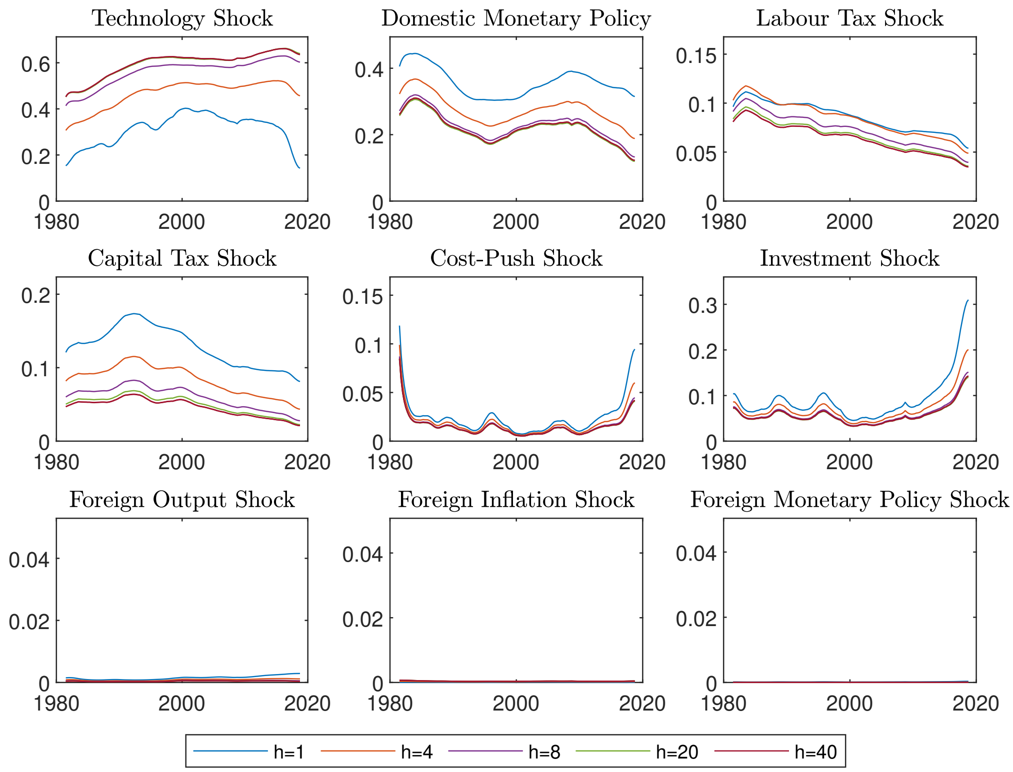

Figure 3: Variance Decomposition for Real GDP Growth for different forecast horizons at quarters $h \in \{1,4,8,20,40\}$ ahead. We deem the structural shocks to technology, cost-push, and investment as (domestic) "market-side shocks" and shocks to monetary, labor-tax and capital-tax policies as "policy-side shocks."

Variation from shock-process volatilities. One shortcoming of using variance decompositions to address our research question is that they only provide details of the contributions of each composite shock, $\tilde{\boldsymbol{u}}_t$. To analyze the relative contribution of *volatility shocks*, it is thus necessary to further decompose the contribution of the two stochastic elements of the structural shocks $\tilde{\boldsymbol{u}}_t$—i.e., changes in the i.i.d. innovations ($\boldsymbol{\varepsilon}_t$) versus changes in the volatility of the distributions of the structural shocks ($\boldsymbol{\Sigma}_t$). With this in mind, we consider the relative share:

$$\xi_{i,t} = \left(\frac{\sigma_{i,t}}{\varepsilon_{i,t}}\right)^2 = \left(\frac{(\sigma_{i,t})^2}{\tilde{u}_{i,t}}\right)^2, \tag{33}$$

where unity is the reference point since this would imply equal contribution of SV and iid shocks.

the variance decompositions across the sample period.

What this refined measure tells us is as follows: A magnitude of $\xi_{i,t} > 1$ implies that the stochastic second moment of a particular shock i is more important in accounting for the variations in the compound structural shock i, i.e., $\tilde{u}_{i,t}$. The complement of this statistic, which accounts for the relative share of the i.i.d. component, can be readily deduced and interpreted as well.

Figure 4 presents the evolution of the relative shares of the structural shocks, as defined in (33), over the entire sample. Here is how one should read this chart: The dashed-black line with a value of unity is our reference point for the ratio defined in (33). Any realization above (below) one would suggest that the SV component is relatively more (not as) dominant than the i.i.d. first-moment component in overall driving a particular structural shock, at each sample period. The blue (solid) line represents the posterior median of the relative shares.[20]

Over the estimated sample period, we see that the SV component was relatively larger during times of economic turmoil. This is seen by large values in the cost-push and monetary policy shocks during the energy crisis of the early 1980s, and the investment shock during the recessions of the early 1980s and 90s, along with the 2000 dotcom crisis and 2007/08 GFC. For example, in 1980, the riskiness of the distribution (i.e., the SV component) of cost-push shock was in the order of 1500 times larger than the first-moment component in accounting for the overall structural cost-push factor. Monetary policy's SV component (i.e., riskiness surrounding monetary policy behavior) accounted for something in the order being twice as important as its first-moment shock.

We also find evidence of volatility being a relatively dominant driver of the monetary policy shock when Canada adopted inflation targeting in the early 1990s, and for taxes when the Federal government reformed income tax in 1987 and the effective rate of capital gains taxation in 2000. In the foreign block we find strong evidence for a relatively large SV contribution during the energy crisis, but then iid shocks become relatively more prevalent during the rest of the sample. We emphasize that this does not mean that the SV component was not at all important during this period (see Figure 1 for discussion of the importance of SV), but rather that the iid components were relatively more important during this period.

8 Conclusion

In this paper, our main goal was to identify and measure which of unexpected variations in international economic stochastic volatility, or domestic economic and policy stochastic volatility, are the more dominant drivers of a small open economy's business cycle.

To answer this question, we extended a version of a well-known small-open-economy DSGE model to allow for the volatilities of the structural disturbances to change over time.

Using this model structure as a yardstick for interpretation and quantification, we identified and accounted for both domestic market-side (i.e., technology, cost-push, and investment) and policy-side (i.e.,

[20] We also have information on the estimated credible set—e.g., the 16th and 84th percentiles of the distribution—that contains the posterior estimates on the respective shares. However, the relatively large upper bound of the distribution results in the median being indistinguishable from the lower bound, despite the two being numerically different. Readers interested in seeing what we mean may find the corresponding figure in our Online Appendix G.

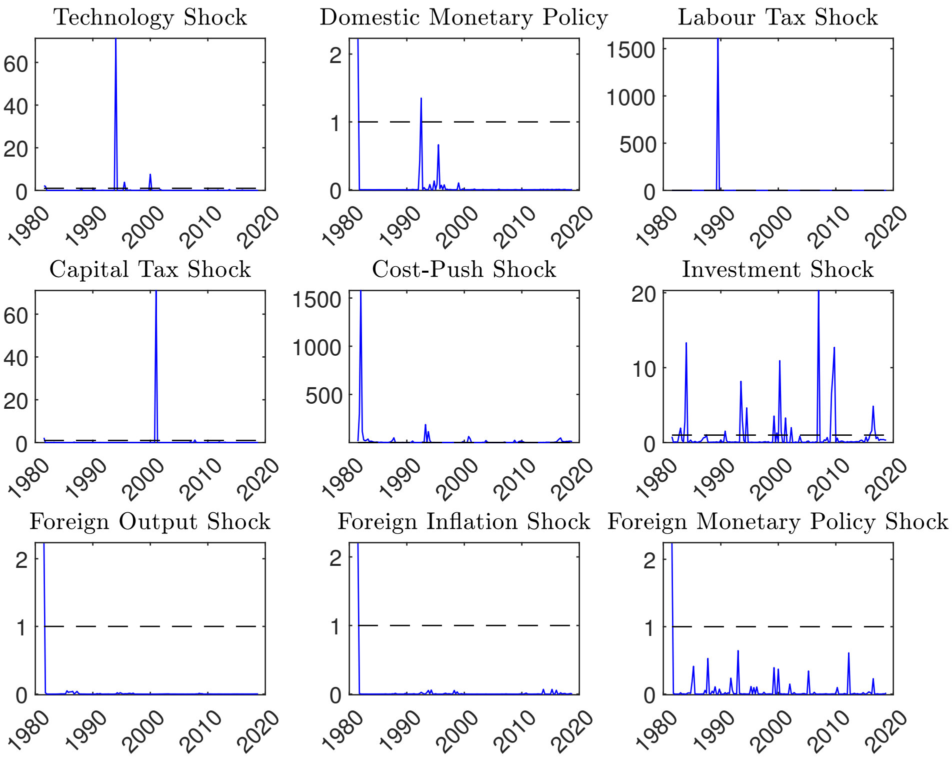

Figure 4: Relative shares of stochastic-volatility in structural shocks

monetary, labor-tax and capital-tax policy) sources of stochastic-volatility shocks, in addition to foreign sources of shocks.

Overall, our shock volatility accounting exercise suggests that the historical movements in the Canadian business cycle are largely due to domestic technology and policy shocks. When further decomposed into their constituent structural i.i.d. and time-varying volatility components, we find that time-varying volatilities in these shocks are relatively more important during times of turmoil. Our model-based shock-volatility accounting attributes (i) the energy crisis of the early 1980s to international, cost-push, and monetary-policy shock volatilities; (ii) the adoption of inflation targeting in the early 1990s to monetary policy uncertainty; (iii) the recessions around the early 1980s and 1990s to investment volatility; and (iv) income-tax and capital-gains tax reform periods to relatively large tax-policy uncertainty shocks in the model.

While the normative question of "what ought policymakers do in response" or "what are optimal policies" is beyond the scope of detailed study here, our estimated shock-volatility accounting suggests that policy design may need to be more gradual and further condition on time-varying volatilities or have explicit policy-simulation models that build in model/shock uncertainty. This is especially pertinent in times of economic turmoil. At least for the estimated historical case of Canada, our results also suggest that one need not worry too much about foreign factors and their riskiness as that is absorbed into existing open-economy channels. A monetary policy that reacts to CPI inflation (a measure that incorporates international relative prices movements) also helps. This conclusion is consistent with recent SVAR evidence for Canada (Ha and So, 2023) and also with earlier optimal policy structural estimation work by

Kam et al. (2009).

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Table 1: Prior and posterior densities for estimated model parameters

Parameter	Description[a]	Family[b]	Prior mean[c]	Prior Std.[c]	Post. Mean[d]	Post. Std.[d]
ρ	Intertemporal ES	G	1.00	0.40	1.00	0.40
η	Elasticity H-F Goods	G	0.90	0.10	0.90	0.10
ϖ	Price-stickiness	N	35.00	15.00	35.82	14.71
ϕ_R	MP, Smoothing	B	0.60	0.10	0.60	0.10
ϕ_Π	MP, Inflation	G	1.80	0.30	1.80	0.30
ϕ_Y	MP, Output	N	0.25	0.13	0.27	0.12
ϕ_W	FP Output (N)	G	0.25	0.13	0.26	0.13
α_W	FP, Smoothing (N)	B	0.60	0.10	0.59	0.10
ρ_A	TFP, Smoothing	B	0.90	0.10	0.90	0.10
κ	Capital Adjustment Stickiness	N	2.00	0.71	2.03	0.69
ϕ_K	FP Output (K)	G	0.25	0.13	0.26	0.13
α_K	FP, Smoothing (K)	B	0.60	0.10	0.60	0.10
ρ_{ϵ_H}	Elasticity H-F Goods, Smoothing (K)	B	0.60	0.10	0.60	0.10
ρ_i	Investment, Smoothing (K)	B	0.60	0.10	0.60	0.10
$\rho_{(Y^*,Y^*,1)}$	VAR-SV, AR	N	0.00	10.00	0.28	0.17
$\rho_{(Y^*,\pi^*,1)}$	VAR-SV, AR	N	0.00	10.00	-0.22	0.22
$\rho_{(Y^*,i^*,1)}$	VAR-SV, AR	N	0.00	10.00	0.26	0.62
$\rho_{(\pi^*,Y^*,1)}$	VAR-SV, AR	N	0.00	10.00	0.27	0.16
$\rho_{(\pi^*,\pi^*,1)}$	VAR-SV, AR	N	0.00	10.00	-0.18	0.22
$\rho_{(\pi^*,i^*,1)}$	VAR-SV, AR	N	0.00	10.00	-0.16	0.60
$\rho_{(i^*,Y^*,1)}$	VAR-SV, AR	N	0.00	10.00	0.01	0.16
$\rho_{(i^*,\pi^*,1)}$	VAR-SV, AR	N	0.00	10.00	0.22	0.21
$\rho_{(i^*,i^*,1)}$	VAR-SV, AR	N	0.00	10.00	0.68	0.59
$\rho_{(Y^*,Y^*,2)}$	VAR-SV, AR	N	0.00	10.00	-0.02	0.15
$\rho_{(Y^*,\pi^*,2)}$	VAR-SV, AR	N	0.00	10.00	-0.10	0.21
$\rho_{(Y^*,i^*,2)}$	VAR-SV, AR	N	0.00	10.00	-0.45	0.58
$\rho_{(\pi^*,Y^*,2)}$	VAR-SV, AR	N	0.00	10.00	0.03	0.15
$\rho_{(\pi^*,\pi^*,2)}$	VAR-SV, AR	N	0.00	10.00	-0.06	0.19
$\rho_{(\pi^*,i^*,2)}$	VAR-SV, AR	N	0.00	10.00	1.17	0.56
$\rho_{(i^*,Y^*,2)}$	VAR-SV, AR	N	0.00	10.00	0.07	0.15
$\rho_{(i^*,\pi^*,2)}$	VAR-SV, AR	N	0.00	10.00	0.03	0.19
$\rho_{(i^*,i^*,2)}$	VAR-SV, AR	N	0.00	10.00	-0.21	0.55
$\sigma_{(\pi^*,Y^*)}$	VAR-SV, COV	N	0.00	10.00	-0.11	0.16
$\sigma_{(i^*,Y^*)}$	VAR-SV, COV	N	0.00	10.00	-0.05	0.15
$\sigma_{(i^*,\pi^*)}$	VAR-SV, COV	N	0.00	10.00	-0.03	0.19
φ_A	log-volatility technology shock, cond. mean	N	0.00	5.00	0.03	1.88
φ_R	log-volatility dom. MP shock, cond. mean	N	0.00	5.00	-0.25	1.95
φ_{τ_W}	log-volatility labor tax shock, cond. mean	N	0.00	5.00	-0.24	1.91
φ_{τ_K}	log-volatility capital tax shock, cond. mean	N	0.00	5.00	-0.15	1.90
φ_{ϵ_H}	log-volatility cost-push shock, cond. mean	N	0.00	5.00	0.00	0.23
φ_μ	log-volatility investment shock, cond. mean	N	0.00	5.00	-0.76	1.43
φ_{y^*}	log-volatility foreign output shock, cond. mean	N	0.00	5.00	-0.19	1.91
φ_{π^*}	log-volatility foreign inflation shock, cond. mean	N	0.00	5.00	-0.27	1.96
φ_{i^*}	log-volatility foreign MP shock, cond. mean	N	0.00	5.00	-0.03	1.93
ψ_A	log-volatility technology shock, AR	TN	0.90	1.00	0.99	1×10^{-4}
ψ_R	log-volatility dom. MP shock, AR	TN	0.90	1.00	0.99	6×10^{-4}
ψ_{τ_W}	log-volatility labor tax shock, AR	TN	0.90	1.00	1.00	3×10^{-4}
ψ_{τ_K}	log-volatility capital tax shock, AR	TN	0.90	1.00	1.00	3×10^{-4}
ψ_{ϵ_H}	log-volatility cost-push shock, AR	TN	0.90	1.00	0.98	182×10^{-4}
ψ_μ	log-volatility investment shock, AR	TN	0.90	1.00	0.98	190×10^{-4}
ψ_{y^*}	log-volatility foreign output shock, AR	TN	0.90	1.00	1.00	19×10^{-4}
ψ_{π^*}	log-volatility foreign inflation shock, AR	TN	0.90	1.00	1.00	8×10^{-4}
ψ_{i^*}	log-volatility foreign MP shock, AR	TN	0.90	1.00	1.00	1×10^{-4}
ω_A	log-volatility technology shock, std. dev.	IG	0.10	0.10	0.10	3×10^{-3}
ω_R	log-volatility dom. MP shock, std. dev.	IG	0.10	0.10	0.01	8×10^{-3}
ω_{τ_W}	log-volatility labor tax shock, std. dev.	IG	0.10	0.10	0.02	4×10^{-3}
ω_{τ_K}	log-volatility capital tax shock, std. dev.	IG	0.10	0.10	0.01	5×10^{-3}
ω_{ϵ_H}	log-volatility cost-push shock, std. dev.	IG	0.10	0.10	0.17	68×10^{-3}
ω_μ	log-volatility investment shock, std. dev.	IG	0.10	0.10	0.08	49×10^{-3}
ω_{y^*}	log-volatility foreign output shock, std. dev.	IG	0.10	0.10	0.03	17×10^{-3}
ω_{π^*}	log-volatility foreign inflation shock, std. dev.	IG	0.10	0.10	0.02	9×10^{-3}
ω_{i^*}	log-volatility foreign MP shock, std. dev.	IG	0.10	0.10	0.01	3×10^{-3}

[a] MP (or FP) stands for Monetary (or Fiscal) Policy rule. TFP denotes Total Factor Productivity. N and K respectively denote labor and capital. AR denotes autoregressive coefficient and COV denotes covariance.

[b] B stands for Beta, G Gamma, IG inverse-Gamma, N Normal and TN Truncated Normal.

[c] Posterior moments are generated from a thinned sample of 10^6 MCMC draws in which we save 1 in 50 draws after a 50,000 draw burn-in. Convergence diagnostics are presented in Appendix E in the online appendix.

— ONLINE APPENDIX —

Volatility Shocks in Markets and Policies:
What Matters for a Small Open Economy like Canada?

Included for reference convenience of the referees/editors

A Model

Here we fill in on the details of the model omitted in the main paper.

A.1 Demand for goods varieties

The aggregate level of consumption is a CES composite index of home and foreign produced consumption goods:

$$C_t = \left[(1-\gamma)^{1/\eta}\left(C_{H,t}^{\frac{\eta-1}{\eta}}\right)+\gamma^{1/\eta}\left(C_{F,t}^{\frac{\eta-1}{\eta}}\right)\right]^{\frac{\eta}{\eta-1}}, \qquad \gamma \in (0,1), \tag{34}$$

in which $\eta > 0$ is the elasticity of substitution between home and foreign goods. Furthermore, these Home and Foreign index goods are Dixit-Stiglitz aggregates over a continuum of differentiated varieties:

$$C_{n,t} = \left[\int_{[0,1]} [C_{n,t}(i)]^{\frac{\epsilon_{n,t}-1}{\epsilon_{n,t}}} \mathrm{d}i\right]^{\frac{\epsilon_{n,t}}{\epsilon_{n,t}-1}},$$

where $n \in \{H,F\}$ and $\epsilon_{n,t} > 1$ is the elasticity of substitution between types of differentiated domestic or foreign goods, which we allow to vary over time and defer details of the law of motion to the next section. Thus, at a given date t, the household faces an associated an expenditure minimization problem, in which they are required to choose varieties of Home and Foreign goods, conditional on their current prices $P_{H,t}$ and $P_{F,t}$. Solving this problem, the optimal consumption demand of each type of good is:

$$C_{H,t} = (1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t, \qquad C_{F,t} = \gamma\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t,$$

where substitution of these demand functions into (34) yields the consumer price index:

$$P_t = \left[(1-\gamma)\,P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{35}$$

Finally, given choices $C_{H,t}$ and $C_{F,t}$, the household chooses varieties $C_{n,t}(i)$, conditional on prices $P_{n,t}(i)$, to minimize the expenditure function:

$$\int_{[0,1]} P_{n,t}(i) C_{n,t}(i)\mathrm{d}i + P_{n,t}\left\{C_{n,t} - \left[\int_{[0,1]} [C_{n,t}(i)]^{\frac{\epsilon_{n,t}-1}{\epsilon_{n,t}}} \mathrm{d}i\right]^{\frac{\epsilon_{n,t}}{\epsilon_{n,t}-1}}\right\}.$$

Solving this problem results in the demand functions and associated aggregate price levels:

$$C_{n,t}(i) = \left(\frac{P_{n,t}(i)}{P_{n,t}}\right)^{-\epsilon_{n,t}} C_{n,t}, \tag{36}$$

$$P_{n,t} = \left(\int_0^1 P_{n,t}(i)^{1-\epsilon_{n,t}}\, di\right)^{\frac{1}{1-\epsilon_{n,t}}}. \tag{37}$$

for all $i \in [0,1]$ and $n \in \{H,F\}$.

A.2 Firms' pricing decision

The decision problem for each firm $i \in [0,1]$ is given by:

$$\Theta_t(i) = \max_{\{P_{H,t+s}(i)\}_{s\in\mathbb{N}}} \left\{ \mathbb{E}_t \sum_{s=0}^{\infty} \mathscr{D}_t \left[\frac{P_{H,t+s}(i)}{P_{H,t+s}} Y_{H,t+s}(i) - \frac{W_{t+s}}{P_{H,t+s}} N_{t+s}(i) \right.\right.$$
$$\left.\left. - AC\left(\frac{P_{H,t+s}(i)}{P_{H,t+s-1}(i)}, Y_{H,t+s}(i) \right) \right] : (11), (12). \text{ and } (13) \right\},$$

where $\mathscr{D}_t = \delta_{t+s} \frac{U_C(C_{t+s},N_{t+s})}{U_C(C_t,N_t)}$ is the stochastic discount factor. The first-order conditions for this problem at every date $t \in \mathbb{N}$ and state $\boldsymbol{s}_t$, imply that firm i's optimal pricing strategy satisfies:

$$\begin{aligned} 0 = (1-\epsilon_{H,t}) \frac{Y_{H,t}(i)}{P_{H,t}} - \frac{MC_t}{P_{H,t}} \frac{\partial Y_{H,t}(i)}{\partial P_{H,t}(i)} - \frac{\partial AC\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)}, Y_{H,t}(i) \right)}{\partial P_{H,t}(i)} \\ - \beta(C_t^a/A_t) \mathbb{E}_t \left\{ \frac{U_C(C_{t+1},N_{t+1})}{U_C(C_t,N_t)} \frac{\partial AC\left(\frac{P_{H,t+1}(i)}{P_{H,t}(i)}, Y_{H,t+1}(i) \right)}{\partial P_{H,t}(i)} \right\}, \end{aligned} \tag{38}$$

where

$$\frac{\partial Y_{H,t}(i)}{\partial P_{H,t}(i)} = -\epsilon_{H,t} \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon_{H,t}-1} \left(\frac{1}{P_{H,t}} \right) Y_{H,t},$$

and,

$$\begin{aligned} \frac{\partial AC\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)}, Y_{H,t}(i) \right)}{\partial P_{H,t}(i)} = \frac{\varpi}{2} AC\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i), Y_{H,t}(i)} \right) \frac{1}{Y_{H,t}(i)} \frac{\partial Y_{H,t}(i)}{\partial P_{H,t}(i)} \\ - \varpi \left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1 \right) \frac{1}{P_{H,t-1}(i)} Y_{H,t}(i) \end{aligned}.$$

The first term on the RHS of (38) is the current real marginal revenue to the firm with respect to its own price variation. The second term is the real marginal cost associated with the marginal variation in labor hiring, as a consequence of the pricing variation's effect on the demand for firm i's output. The third and fourth term, respectively, give current and (expected) future marginal effects of the pricing strategy variation on the firm's profit via the price-adjustment cost terms.

B Stationary RCE characterization

We transform the necessary conditions from Definition 1 into stationary form as follows, where a variable with a "tilde" denotes a stationarized ratio between its original level and the level of domestic total factor productivity, i.e., $\tilde{X}_t := X_t/A_t$.

Household optimal portfolio choices:

$$\tilde{C}_t^{-\rho} = R_t \mathbb{E}_t \left\{ \beta\left(\tilde{C}_t\right) \frac{\tilde{C}_{t+1}^{-\rho}}{\Pi_{t+1}} \exp\left[g_A + \sigma_A \varepsilon_{A,t+1}\right]^{-\rho} \right\}, \tag{39a}$$

$$\tilde{C}_t^{-\rho} = \mathbb{E}_t\left\{\tilde{R}^*_{t+1}\beta\left(\tilde{C}_t\right)\frac{\tilde{C}_{t+1}^{-\rho}Q_{t+1}}{Q_t}\exp\left[g_A+\sigma_A\varepsilon_{A,t+1}\right]^{-\rho}\right\}. \tag{39b}$$

Capital accumulation equation and convex adjustment costs:

$$\tilde{K}_{t+1}\exp(g_{A,t+1}) = (1-\xi)\tilde{K}_t + \mu_t\tilde{I}_t\left[1-\mathscr{D}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\right)\right], \tag{39c}$$

$$\mathscr{D}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\exp(g_{A,t+1})-\exp(g_A)\right), \tag{39d}$$

Investment condition:

$$\frac{q_{N,t}}{P_t} = \delta_t\mathbb{E}_t\left\{\frac{\tilde{C}_{t+1}^{-\rho}}{\tilde{C}_t^{-\rho}}\exp\left(g_{A,t+1}\right)\left((1-\xi)\frac{q_{N,t+1}}{P_{t+1}}+\left(1-\tau_{K,t+1}\right)\alpha\frac{\tilde{Y}_{H,t+1}}{\tilde{K}_{t+1}}\frac{mc_{H,t+1}}{p_{H,t+1}}+\tau_{K,t+1}\xi\right)\right\}. \tag{39e}$$

Firm optimal pricing and hiring:

$$\begin{aligned}\Pi_{H,t}(\Pi_{H,t}-\Pi)-\frac{\epsilon_H}{2}(\Pi_{H,t}-\Pi)^2 &= \frac{\epsilon_H}{\theta}\left[mc_{H,t}-\frac{\epsilon_H-1}{\epsilon_H}\right]\\ &+\beta(\tilde{C}_t^a)\mathbb{E}_t\left\{\frac{\tilde{C}_{t+1}^{-\rho}}{\tilde{C}_t^{-\rho}}\left(\Pi_{H,t+1}-\Pi\right)\Pi_{H,t+1}\cdot\frac{\tilde{Y}_{H,t+1}}{\tilde{Y}_{H,t}}\exp\left[g_A+\sigma_A\varepsilon_{A,t+1}\right]^{1-\rho}\right\},\end{aligned} \tag{39f}$$

$$\tilde{Y}_{H,t} = N_t^{1-\alpha}\tilde{K}_t^{\alpha}. \tag{39g}$$

Labor market and goods market clearing:

$$\psi N_t^{\varphi}\tilde{C}_t^{\rho} = (1-\tau_{W,t})mc_{H,t}p_{H,t} \tag{39h}$$

$$\left[1-\left(\Pi_{H,t}-\Pi\right)^2\right]\tilde{Y}_t = \tilde{Y}_{H,t} \equiv \left(p_{H,t}\right)^{-\eta}\left[(1-\gamma)(\tilde{C}_t+\tilde{I}_t+\tilde{G}_t)+\gamma Q_t^{\eta}\tilde{C}_t^*\right]. \tag{39i}$$

Government budget constraint:

$$\tilde{G}_t = \left((1-\alpha)\tau_{W,t}+\alpha\tau_{K,t}\right)mc_{H,t}\tilde{Y}_{H,t}, \tag{39j}$$

Identities:

$$p_{H,t} = \left[\frac{1-\gamma\left(Q_t\right)^{1-\eta}}{1-\gamma}\right]^{\frac{1}{1-\eta}}, \tag{39k}$$

$$\Pi_t = \Pi_{H,t} \times \frac{p_{H,t-1}}{p_{H,t}}. \tag{39l}$$

Note that $mc_{H,t} := MC_t/P_{H,t}$, N_t, $p_{H,t} := P_{H,t}/P_t$, Q_t, Π_t and $\Pi_{H,t}$ are already stationary variables, as are R_t, $\tilde{R}^*_t$, $\tau_{W,t}$ and $\tau_{K,t}$.

Given the exogenous stochastic process s_t, and policy behaviors, (18), (20), and stochastic volatility processes, (27), (29) and (31), the system above characterizes a bounded stochastic process for allocation $\{\tilde{C}_t, N_t, \tilde{I}_t, \tilde{G}_t, \tilde{Y}_{H,t}, mc_{H,t}\}_{t\in\mathbb{N}}$ and pricing functions $\{\Pi_{H,t}, p_{H,t}, \Pi_t, Q_t\}_{t\in\mathbb{N}}$.

B.1 Steady state and model calibrations

In this section we describe the model's non-stochastic steady state (hereinafter we will refer to this as the unique "steady state"). It is easy to check that the steady state results in an under identified system. Thus, to get a unique solution, we need to use a combination moment matching to first-order observable (i.e., long-run) data as well as calibration. In what follows, we denote a variable without explicit time subscript as its steady state point.

First, given the data's long-run foreign-output and consumption shares, respectively, $\tilde{C}^*/\tilde{Y} := \tilde{Y}^*/\tilde{Y}$ and $\tilde{C}/\tilde{Y}$, we estimate (pin down) the share γ, from matching the first moment using (39i).

Next, Equation (39f), implies that the steady state marginal cost is given by $mc_H = (\epsilon_H - 1)/\epsilon_H$. Thus, given parameter estimates (ρ, φ) we can choose ψ in (39h) to calibrate the proportion of hours worked to $N = 0.33$. Given N and mc_H, the steady state level of capital can then be pinned down from (13) and (14). These two inputs are then pin down $\tilde{Y}_H$ in (39g), which is subsequently used in the first equality of (39i) to pin down Y. Domestic consumption and foreign output are then derived by multiplying the implied steady state level of output by the first moment statistics in the respective time series. Steady state capital and labor tax are similarly pinned down by their respective first moment statistics from our derived data (0.32 and 0.21 respectively). Given the steady state capital and labor tax rates, we then choose α to match government expenditure share of output in (39j) to it's observed first moment of 0.21 - this equates to $\alpha = 0.31$. The stationary capital accumulation equation (39c) is then used to pin down steady state investment.

Next, we normalize the steady state real exchange rate to unity (i.e. $Q = 1$). From (39k), this implies that $p_H = 1$, and from (39l), we will also have $\Pi = \Pi_H$. To pin down the steady state level of inflation we first pin down both domestic and foreign real gross interest rates (i.e., R and R^* by making use of first moment statistics from their respective time series data. More precisely, using quarterly data on Canada's bank rate we set $R = 1.02$. Similarly, using quarterly data on the federal funds rate and US CPI inflation rates, the steady state international nominal interest and inflation rates are respectively set to $i^* = 1.27$ and $\pi^* = 0.67$ percent. Using the well known Fisher relationship, these values imply a real gross international interest rate of $R^* = 1.00596$. Using these two results along with the steady state domestic and foreign Euler conditions (39a) and (39b), implies that $\Pi = \frac{R}{R^*}$. Similarly, (39b) implies that $\delta = \frac{g_A}{R^*}$, conditional on g_A. These values can then be used to solve for the steady state value of ζ in the endogenous discount factor (where $\vartheta \approx 0$ is a calibrated value).

C Data

All data is quarterly and the period under investigation is 1981:Q1-2018:Q4.

Domestic macroeconomic variables. Canadian GDP, the implicit price deflator, civilian population and CPI data are sourced from the Canadian National Statistical Agency (Tables: 36-10-0104-01, 36-10-0106-01, 17-10-0009-01 and 18-10-0004-01). Per capita growth rate of real GDP was calculated by first taking the ratio of the nominal series to the deflator, then dividing the result by the total population, and computing the log-difference of the resulting quarterly series. Inflation is simply the log-difference of the implied quarterly series after taking the appropriate three month average. The nominal interest and exchange rate data was sourced from the FRED Database maintained by the St.Louis Fed (series IRSTCB01CAM156N and DEXCAUS). We transform the interest rate series by converting to gross rates and taking the (natural) log. The nominal exchange rate is converted to a real rate by multiplying by the ratio of the Canadian to US GDP deflator series. We then convert the series to growth rates by taking the log difference. Before entering the model, we subtract the implied the long-run first moment from each series. In addition to these observable series we also use data from World Bank to pin down consumption and government expenditure shares of output (Household final consumption expenditure and General government final consumption expenditure respectively (% of GDP 1981-2018.)).

Foreign macroeconomic variables. US real GDP per capita, GDP deflator and nominal interest rate data were sourced from the FRED Database maintained by the St.Louis Fed (series A939RX0Q048SBEA, GDPCTPI and FEDFUNDS respectively). Real GDP growth and inflation were respectively calculated as the log-difference of the real GDP and CPI series, while the monthly nominal interest rate was converted to a quarterly rate by geometric mean and dividing by 400. Before entering the model each series was demeaned by its implied long-run first moment. To account for the zero lower bound, we use the shadow rate from Wu and Xia (2016) during the Great Recession which is available from Jing Wu's website. Recently developed alternative shadow rates that discuss the issues of policy at the ZLB include Lombardia and Zhub (2018) and Ikeda et al. (2020).

Labor income tax rates. Following Born and Pfeifer (2014) and Fernández-Villaverde et al. (2015), our approach to calculating an average tax rates closely follows the works of Mendoza et al. (1994), Jones (2002), and Leeper et al. (2010). For completeness, we list the details of this two-step procedure. In the first instance, average personal income tax is computed as:

$$\tau_p = \frac{PIT}{WS + PI + RI + CP + NFI}, \tag{40}$$

where PIT denotes the level of (aggregate) personal income tax, WS is income from wage and salaries, PI is property income, RI is rental income, CP is corporation profits and NFI is income from non-farm entities. In our calculations we exclude property taxes due to a lack of available data. Next, given (40), the average labor tax rates are computed as:

$$\tau_N = \frac{\tau_p WS + SS}{CE}, \tag{41}$$

where SS is the total Social Security benefits and CE is the Compensation for Employees. Finally, given (40), the average capital tax rates are computed as:

$$\tau_K = \frac{\tau_p CI + CT}{CI}, \tag{42}$$

All data was sourced from the Canadian National Statistical Agency (Tables: 36-10-0112-01 and 36-10-0114-01). We note that the average tax rates enter the model in demeaned (natural) logs. The resulting series are plotted Figure 5.

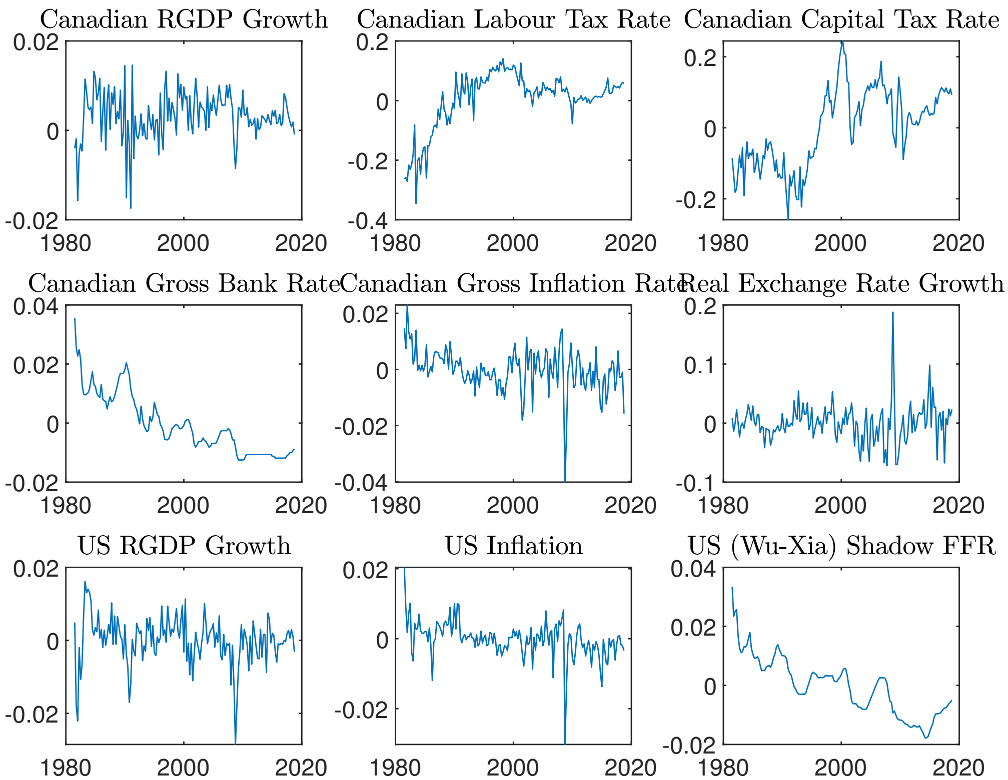

Figure 5: Observed Data

D The Estimation Algorithm

D.1 Standard DSGE Model with Homoskedastic Disturbances

To fix ideas, we begin with the standard linear DSGE framework and discuss its Bayesian estimation problem. To draw from the posterior distribution of the standard DSGE model's parameters—i.e., the model without stochastic volatility—we can follow the algorithm set out in Appendix A of Justiniano and Primiceri (2008).

Let the vector $\boldsymbol{\theta}^{(g)}$ denote the saved draw of all parameters of the baseline DSGE model at iteration $g > 0$. Using Dynare (version 4.4.3), the first-order Taylor approximation of the stationary RCE conditions has a linear Gaussian state-space representation:

$$\mathbf{y}_t^o = \mathbf{H}^o \mathbf{x}_t, \tag{43}$$

$$\mathbf{x}_t = \mathbf{A}\left(\boldsymbol{\theta}^{(g)}\right)\mathbf{x}_{t-1} + \mathbf{B}\left(\boldsymbol{\theta}^{(g)}\right)\mathbf{u}_t, \tag{44}$$

where $\mathbf{y}_t^o$ is a $N_Y \times 1$ vector of observable variables, $\mathbf{x_t}$ is a $N_x \times 1$ vector of endogenous/state variables in log-deviation from the deterministic steady state, $\mathbf{H}^o$ is a $N_Y \times N_x$ selection matrix that maps the data to their model counterpart, $\mathbf{A}\left(\boldsymbol{\theta}^{(\mathbf{g})}\right)$ and $\mathbf{B}\left(\boldsymbol{\theta}^{(\mathbf{g})}\right)$ are respectively $N_x \times N_x$ and $N_x \times N_u$ matrices that contain (implicit) cross-equation restrictions involving the model's deep parameters, and $\mathbf{u}_t \sim N(\mathbf{0}, \Sigma)$ is a $N_u \times 1$ vector of (independent) structural shocks.

Since the posterior distribution of the DSGE models parameters does not belong to a standard class of distributions, we follow the now standard practice of implementing a random walk Metropolis-Hastings (RW-MH) MCMC procedure through which a new candidate parameter vector; $\theta^{(c)}$, is drawn from a proposal density, and accepted with probability:

$$\alpha = \min\left\{1, \frac{\mathscr{L}\left(\mathbf{Y}|\theta^{(c)}\right) p\left(\theta^{(c)}\right)}{\mathscr{L}\left(\mathbf{Y}|\theta^{(g)}\right) p\left(\theta^{(g)}\right)}\right\}, \tag{45}$$

where $\mathbf{Y} := \{\mathbf{y}_t^o\}_{t=1}^T$ is the matrix of data, $\mathscr{L}\left(\mathbf{Y}|\theta^{(i)}\right)$ is the model likelihood and $p\left(\theta^{(i)}\right)$ is the prior distribution where $i \in \{c, g\}$. If $\theta^{(c)}$ is accepted then $\theta^{(g+1)} = \theta^{(c)}$, otherwise $\theta^{(g+1)} = \theta^{(g)}$.[21]

D.2 Modeling Heteroskedastic Disturbances

As discussed in Appendix B of Justiniano and Primiceri (2008), when the structural shocks exhibit heteroskedastic disturbances in the form of latent stochastic volatilities, then the above algorithm must be modified from a Metropolis MCMC to a Metropolis-within-Gibbs MCMC algorithm. To this end, equation (44) is augmented as:

$$\mathbf{x}_t = \mathbf{A}\left(\boldsymbol{\theta}^{(g)}\right)\mathbf{x}_{t-1} + \mathbf{B}\left(\boldsymbol{\theta}^{(g)}\right)\tilde{\mathbf{u}}_t, \tag{46}$$

where $\mathbf{x}_t$, $\mathbf{A}\left(\boldsymbol{\theta}^{(\mathbf{g})}\right)$ and $\mathbf{B}\left(\boldsymbol{\theta}^{(\mathbf{g})}\right)$ are as defined in the previous section and $\tilde{\mathbf{u}}_t$ is a $N_u \times 1$ vector of structural shocks with a time-varying covariance matrix, $N_u = \text{card}(I)$ and I is defined as the finite set of structural shock indexes in the paper. Let each of the structural shocks be indexed by i, the associated i stochastic volatilities are modeled as a non-linear state space model with measurement and state equations respectively defined by a random-walk-plus-noise model:

$$\tilde{u}_{i,t} = \sigma_{i,t}\varepsilon_{i,t}, \tag{47}$$

$$h_{i,t} = h_{i,t-1} + \nu_{i,t}, \tag{48}$$

where $h_{i,t} = \log\sigma_{i,t}$, $\varepsilon_{i,t} \sim N(0,1)$ and $\nu_{i,t} \sim N\left(0, \omega_i^2\right)$ for $i = 1, \ldots, N_u$. In what follows we simplify the notation by letting $\tilde{\mathbf{u}} = (\tilde{\mathbf{u}}_1, \ldots, \tilde{\mathbf{u}}_T)$, $\mathbf{h}_i = \left(h_{i,1}, \ldots, h_{i,T}\right)'$, $\mathbf{H} = \left(\mathbf{h}_1, \ldots, \mathbf{h}_{N_u}\right)$, and $\boldsymbol{\omega} = \left(\omega_1^2, \ldots, \omega_{N_u}^2\right)$. To illustrate

[21] For a textbook treatment of the RW-MH algorithm in DSGE models see Chapter 4 of Herbst and Schorfheide (2015).

the Metropolis-within-Gibbs MCMC algorithm, let $\boldsymbol{\theta}^{(g)}$,$\mathbf{H}^{(g)}$ and $\boldsymbol{\omega}^{(g)}$ denote the last saved draw of all parameters of the baseline DSGE model, stochastic volatilities and associated parameters. Estimation of the DSGE model parameters, stochastic volatilities and associated parameters in iteration $(g+1)$ involves the following five steps.

D.2.1 Draw the structural shocks

In order to get a draw of the stochastic volatilities we must first obtain a sample of the structural shocks, $\tilde{\mathbf{u}}^{(g+1)}$, associated with the approximate solution of the model (46). This is completed with the efficient disturbance simulation smoother as developed by Durbin and Koopman (2002).

D.2.2 Draw the stochastic volatilities

Conditional on $\tilde{\mathbf{u}}^{(g+1)}$, the associated stochastic volatilities can be estimated with the two stage auxiliary mixture sampling approach developed by Kim et al. (1998). In the first stage, equation (47) can be made linear in $\sigma_{i,t}$ by first squaring both sides and then taking the logarithm:

$$\tilde{u}^*_{i,t} \quad = \quad 2h_{i,t} + \tilde{\varepsilon}^*_{i,t}, \tag{49}$$

where $\tilde{u}^*_{i,t} = \log\left(\tilde{u}^2_{i,t}\right)$ and $\tilde{\varepsilon}^*_{i,t} = \log\left(\varepsilon^2_{i,t}\right)$. In practice $\tilde{u}^*_t = \log\left(\tilde{u}^2_{i,t} + c_1\right)$ where c_1 is a small constant that makes the estimation procedure more robust - in practice it is common to set $c_1 = 10^{-4}$. The cost of linearizing the measurement equation in this manner is that the innovations are no longer Gaussian, but instead follow a $\log-\chi^2_1$ distribution. This means that standard estimation algorithms for linear Gaussian state space models can not be directly applied.

To overcome this computational difficulty, the second step in the auxiliary mixture sampling approach is make the transformed measurement equation conditionally Gaussian by defining a (conditionally) Gaussian auxiliary mixture random variable that matches the moments of the $\log-\chi^2_1$ distribution. More precisely, Kim et al. (1998) show that:

$$f\left(\tilde{\varepsilon}^*_{i,t}\right) \quad \approx \quad \sum_{j=1}^{7} p_j f\left(\tilde{\varepsilon}^*_{i,t} | s_{i,t} = j\right), \tag{50}$$

where $s_{i,t} \in \{1,\dots,7\}$ is an auxiliary random variable that serves as the mixture component indicator for the ith innovation at date t, $p_j = Pr\left(s_i = j\right)$ and $f(\cdot)$ is a (conditional) Gaussian density with mean $m_j - 1.2704$ and variance r^2_j. For completeness, the parameters for each of the seven Gaussian distributions in (10) are reported in Table 2.

Note that the Gaussian mixture does not have any unknown parameters. Thus, conditional on $\mathbf{s}^{(g)} = \left(\mathbf{s}_1,\dots,\mathbf{s}_{N_u}\right)$, where $\mathbf{s}_i = (s_1,\dots,s_T)'$, the system has an approximate linear Gaussian state space form, from which a new draw $\mathbf{H}^{(g+1)}$ can be obtained with standard linear Gaussian sampling algorithms. In our application $\mathbf{H}^{(g+1)}$ are efficiently sampled with the precision-sampling based algorithm explained in Chan and Hsiao (2014).

Table 2: A seven-component Gaussian mixture for approximating the $\log \chi_1^2$ distribution

Component	p_j	m_j	r_j^2
1	0.00730	-10.12999	5.79596
2	0.10556	-3.97281	2.61369
3	0.00002	-8.56686	5.17950
4	0.04395	2.77786	0.16735
5	0.34001	0.61942	0.64009
6	0.24566	1.79518	0.34023
7	0.25750	-1.08819	1.26261

D.2.3 Draw the indicators of the mixture approximation

Given the draws $\tilde{\mathbf{u}}^{(g+1)}$ and $\mathbf{H}^{(g+1)}$, the components of the auxiliary mixture component indicator: $s_{i,t}^{(g+1)}$, can be independently sampled from the following seven point discrete distribution:

$$Pr\left(s_{i,t}^{(g+1)} = j|\tilde{u}_{i,t}^{(g+1)}, h_{i,t}^{(g+1)}\right) = \frac{1}{c_t} p_j f\left(\tilde{\varepsilon}_{i,t}^{*}|2h_{i,t}^{(g+1)} + m_j - 1.2704, r_j^2\right), \tag{51}$$

where $c_t = \sum_{j=1}^{7} p_j f\left(\tilde{\varepsilon}_{i,t}^{*}|2h_{i,t}^{(g+1)} + m_j - 1.2704, r_j^2\right)$ is a normalization constant.

D.2.4 Draw the associated parameters of the stochastic volatilities

Having generated $\tilde{\mathbf{u}}^{(g+1)}$ and $\mathbf{H}^{(g+1)}$, elements of the vectors $\boldsymbol{\varphi}^{(g+1)}$, $\boldsymbol{\psi}^{(g+1)}$ and $\boldsymbol{\omega}^{(g+1)}$ can be sampled with usual distributions as in, e.g. Chan and Hsiao (2014). In particular, consider the following independent prior distributions:

$$\varphi_i \sim N(\varphi_0, V_\varphi), \quad \psi_i \sim N(\psi 0, V_\psi)1(|\psi| < 1), \quad \omega_i^2 \sim IG\left(\nu_{\omega_i^2}, S_{\omega_i^2}\right). \tag{52}$$

Elementary operations show that the associated conditional posterior distributions are

$$\left(\varphi_i{}^{(g+1)}|\tilde{u}_{i,t}^{(g+1)}, h_{i,t}^{(g+1)}, \psi_i{}^{(g+1)}, \omega_i^{2(g+1)}\right) \sim N\left(\hat{\mu}_\varphi, D_\varphi\right) \tag{53}$$

$$\left(\omega_i^{2(g+1)}|\tilde{u}_{i,t}^{(g+1)}, \sigma_{i,t}^{(g+1)}, \varphi_i{}^{(g+1)}, \psi_i{}^{(g+1)}\right) \sim IG\left(\nu_{\omega_i^2} + \frac{T-1}{2}, S_{\omega_i^2} + \sum_{t=2}^{T}(h_t - h_{t-1})^2\right) \tag{54}$$

where $D_\varphi = \left(V_\varphi^{-1} + \mathbf{X}_\varphi' \boldsymbol{\Sigma}_\omega^{-1} \mathbf{X}_\varphi\right)^{-1}$ and $\hat{\mu}_\varphi = D_\varphi\left(V_\varphi^{-1}\varphi_0 + \mathbf{X}_\varphi' \boldsymbol{\Sigma}_\omega^{-1} \mathbf{z}_\varphi\right)$ in which $\mathbf{X}_\varphi = (1-\psi_i, \dots, 1-\psi_i)'$, $\boldsymbol{\Sigma}_\omega = (\omega_i^2, \dots, \omega_i^2)$ and $\mathbf{z}_\varphi = (h_1 - \psi, \dots, h_T - \psi)'$. Finally, note that the truncated normal prior will produce a conditional posterior such that

$$\left(\psi_i{}^{(g+1)}|\tilde{u}_{i,t}^{(g+1)}, h_{i,t}^{(g+1)}, \varphi_i{}^{(g+1)}, \omega_i^{2(g+1)}\right) \propto p(\psi_i) g(\psi_i) e^{-\frac{1}{2\omega_i^2}\sum_{t=2}^{T}(h_t - \varphi_i - \psi_i(h_{t-1} - \varphi_i))^2}, \tag{55}$$

where $p(\psi_i)$ is the truncated normal prior given above and $g(\psi_i) = (1-\psi_i^2)^{1/2}\exp - \frac{1}{2\omega_i^2}(h_1 - \mu_h)^2$, which is a non-standard distribution. Following Chan and Hsiao (2014), we implement an independence-chain Metropolis-Hastings step with proposal distribution $N(\hat{\psi}_i, D_{\psi_i})1(|\psi_i < 1|)$, where $D_{\psi_i} = (V_{\psi_i}^{-1} +$

$\mathbf{X}'_{\psi}\mathbf{X}_{\psi}/\omega_i^2)^{-1}$ and $\hat{\psi}_i = D_{\psi_i}(V_{\psi}^{-1}\psi_0 + \mathbf{X}_{\psi}\mathbf{z}_{\psi}/\omega_i^2)^{-1})$, with $\mathbf{X}_{\psi} = (h_1 - \varphi, \ldots, h_{T-1} - \varphi)'$ and $\mathbf{z}_{\psi} = (h_2 - \varphi, \ldots, h_T - \varphi)'$.

D.2.5 Draw the DSGE parameters

Finally, as in the baseline model, the DSGE model parameters are sampled using a random walk Metropolis MCMC procedure in which the new candidate parameter vector $\theta^{(c)}$ is drawn from a proposal density. Since the model now includes stochastic volatility the likelihood ratio must be adjusted such that $\theta^{(c)}$ is now accepted with probability:

$$\alpha = \min\left\{1, \frac{\mathscr{L}\left(\mathbf{Y}|\theta^{(c)}, \mathbf{H}^{(g+1)}\right) p\left(\theta^{(c)}\right)}{\mathscr{L}\left(\mathbf{Y}|\theta^{(g)}, \mathbf{H}^{(g+1)}\right) p\left(\theta^{(g)}\right)}\right\} \tag{56}$$

where $\mathbf{Y}$ is the matrix of data, $\mathscr{L}\left(\mathbf{Y}|\theta^{(j)}, \mathbf{H}^{(g+1)}\right)$ is the model likelihood and $p\left(\theta^{(j)}\right)$ is the prior distribution where $i \in \{c, g\}$. If $\theta^{(c)}$ is accepted then $\theta^{(g+1)} = \theta^{(c)}$, otherwise $\theta^{(g+1)} = \theta^{(g)}$.

D.3 Foreign VAR-SV process

Since the DSGE-SV model has many parameters, we reduce the computational burden by estimating the autoregressive coefficients and contemporaneous covariance terms in the foreign VAR-SV process externally to the DSGE-SV model. These parameters are then set equal to their resulting posterior mean values when estimating the DSGE model. Since estimation of VAR-SV models is standard we here save space and refer readers interested in the details to Chan and Eisenstat (2017).

E Convergence Diagnostics

To assess convergence of the Markov chain to its ergodic distribution we conduct both formal and informal diagnostic checks. All test statistics were computed with a thinned sample of 10^6 MCMC draws in which 1 in 100 draws was stored after a 50,000 draw burn-in. As an informal check Figures 6 and 7 show the stored MCMC draws of the DSGE micro-parameters and exogenous processes, respectively, after thinning the chain. Since the draws resemble a white noise process, there is informal evidence to support the hypothesis that the draws are in fact independent. Next, Figure 8 shows the inefficiency factors (IFs) for both the DSGE micro-parameters and the log-volatilities. The IF is the inverse of the well known relative numerical efficiency (RNE) measure of Geweke (1992). In each case, the IFs are computed by comparing the first 10% of draws to the final 50%. For interpretation purposes IFs of approximately 20 or less are indicative of convergence. Overall, the evidence suggests that the Markov chains for each parameter have converged to their stationary distributions.

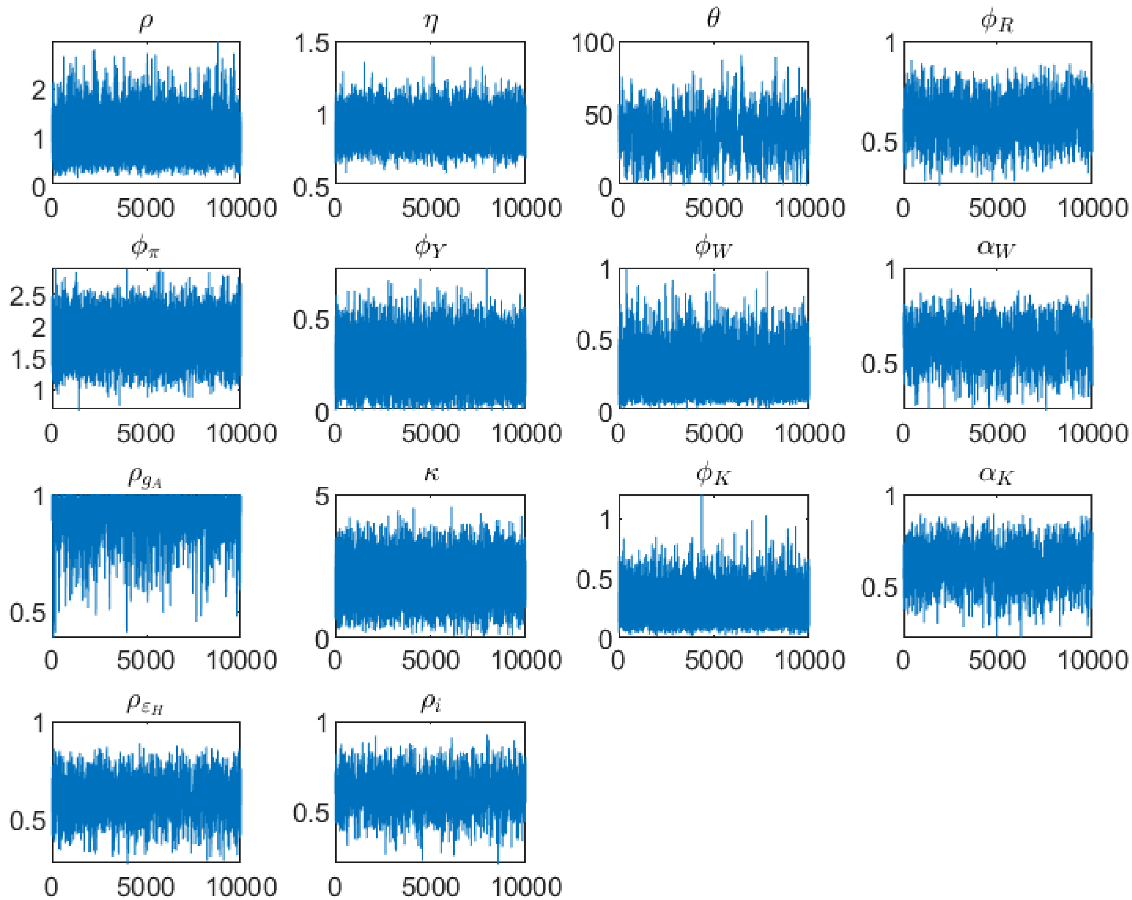

Figure 6: Markov chain of DSGE-SV micro parameters

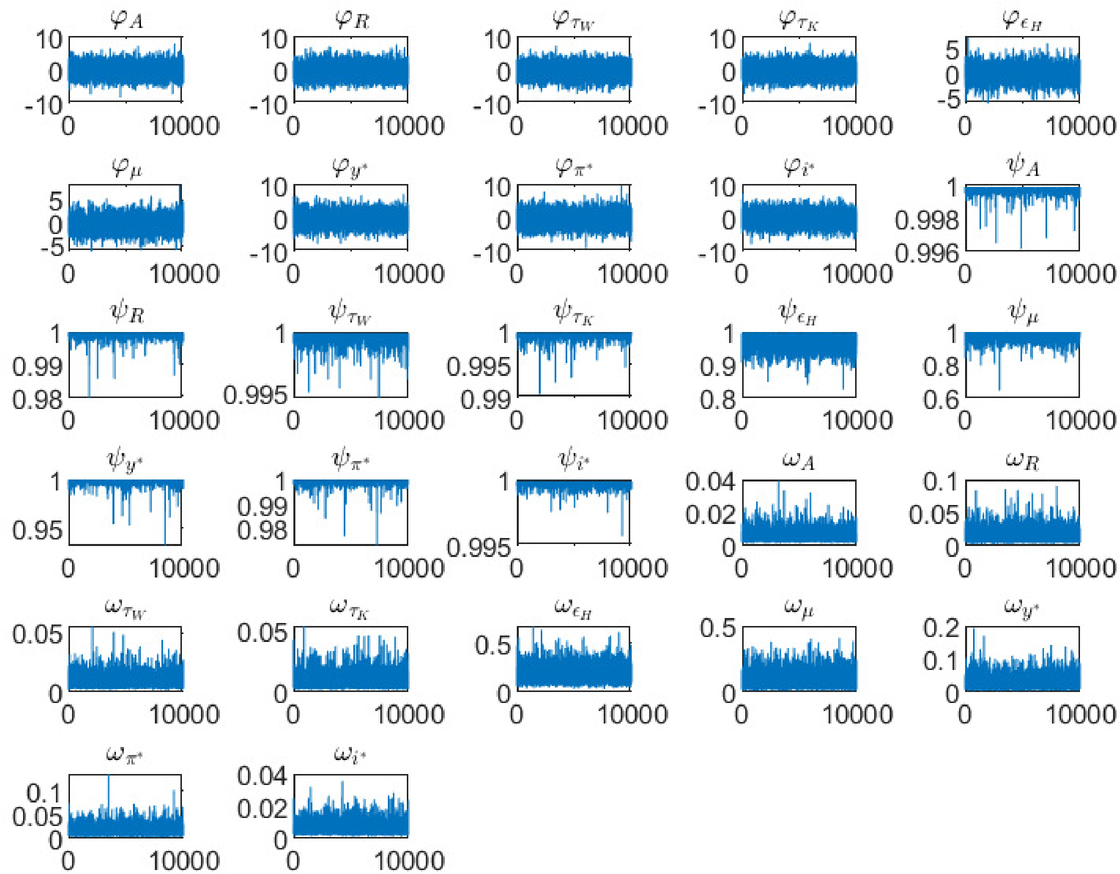

Figure 7: Markov chain of DSGE-SV parameters associated with state equations of log-volatilities

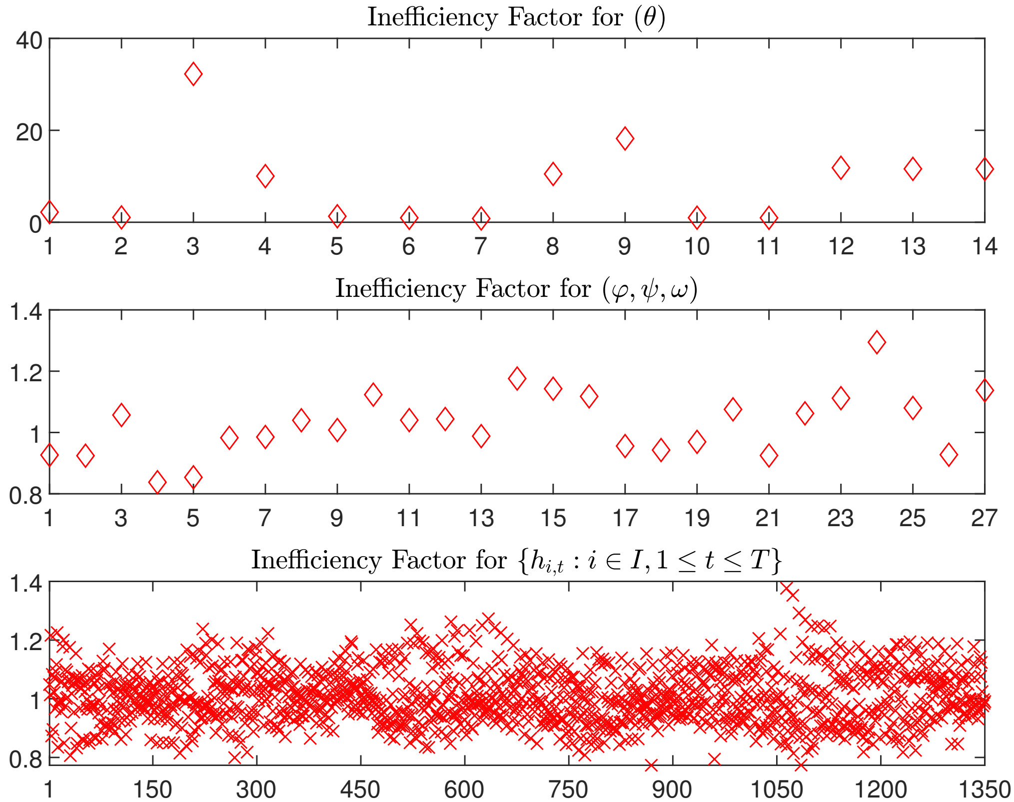

Figure 8: Parameter Markov Chain Convergence Statistics. The top panel reports the IF statistics each of the 9 the DSGE-SV micro parameters, $\boldsymbol{\theta}$. The middle panel reports the IF statistics for the 27 parameters associated with 9 state equations of log-volatilities $(\boldsymbol{\varphi}, \boldsymbol{\psi}, \boldsymbol{\omega})$. The bottom panel reports that of the 9 stochastic volatilities over a length-T ($T = 150$) sample history, $\mathbf{H}$.

For completeness, we also report the same convergence diagnostics for the associated VAR-SV parameters. Here we used a complete set of 10,000 draws after a burn-in of 5,000 draws. Figure 9 contains the stored MCMC draws of the AR coefficients and contemporaneous covariance terms. Figure 10 contains the inefficiency factors for the same parameters. As in the case of the DSGE-SV parameters, evidence again suggests that the Markov chains for each parameter have converged to their stationary distributions.

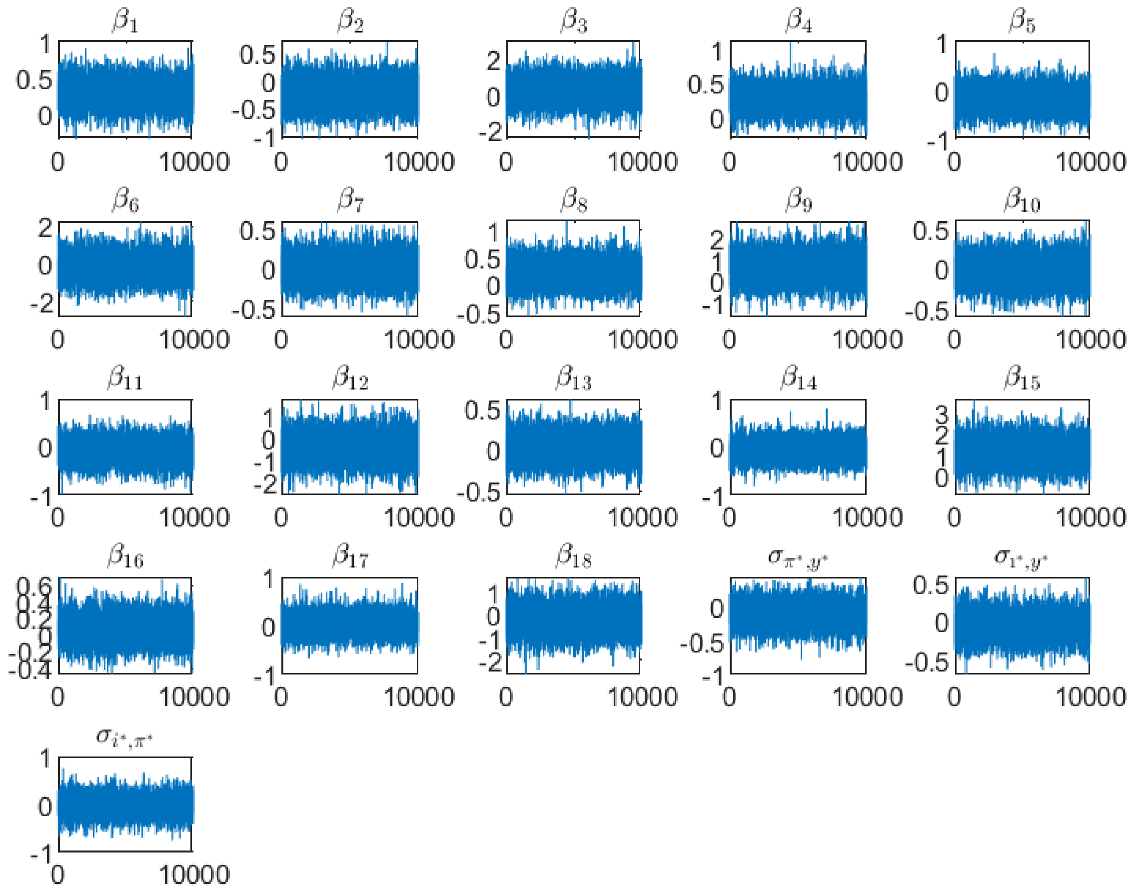

Figure 9: Markov chain of VAR-SV parameters

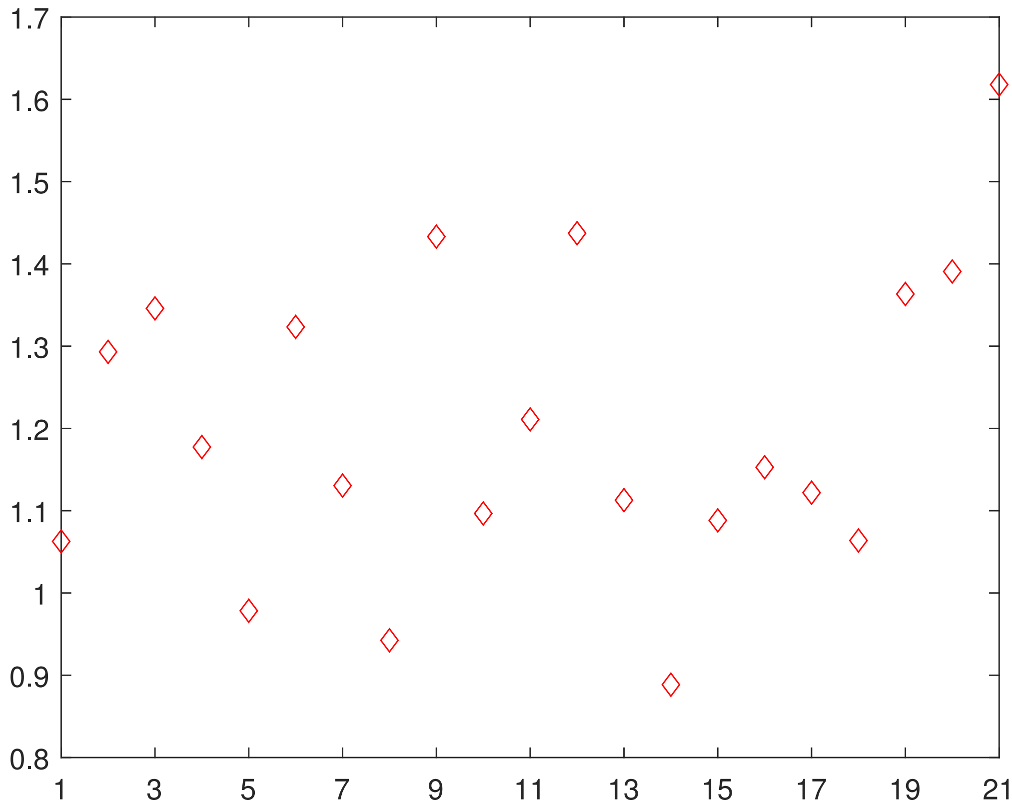

Figure 10: Parameter Markov Chain Convergence Statistics for the VAR-SV parameters: 18 AR parameters and 3 contemporaneous covariance terms

F International correlations

Figure 11 contains empirical international correlations between Canadian and U.S. series observed in the data (line with asterisks) and produced by the model with parameters evaluated at the posterior mean (line with circles). We find that the model often underestimates the international correlations. This is a well known problem in small open economy models which arises possibly due to incomplete theories about the determinants of international spillovers. For instance, using data on Canada and the US, Justiniano and Preston (2010a) find that model-implied international cross-correlations are essentially zero. This is also what we observe in Figure 11.

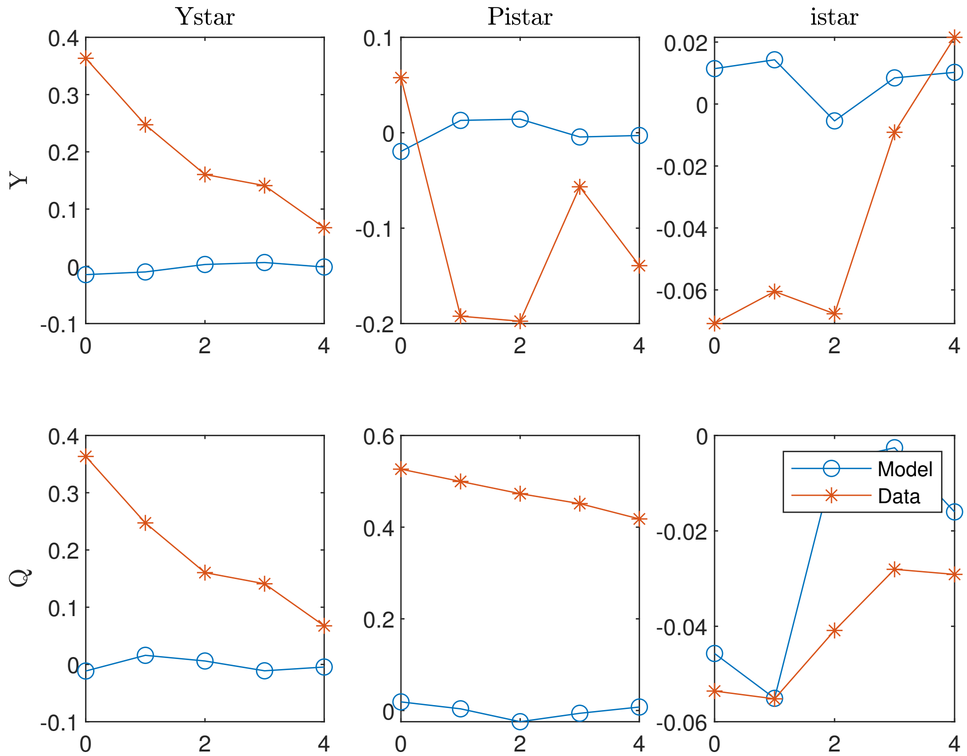

Figure 11: International correlations between domestic output (Y) and the real exchange rate (Q) and foreign (U.S.) output (Ystar), inflation (Pistar) and interest rate (istar). The horizontal axis measure the lag interval in the correlation statistics.

G Additional Figure and Tables

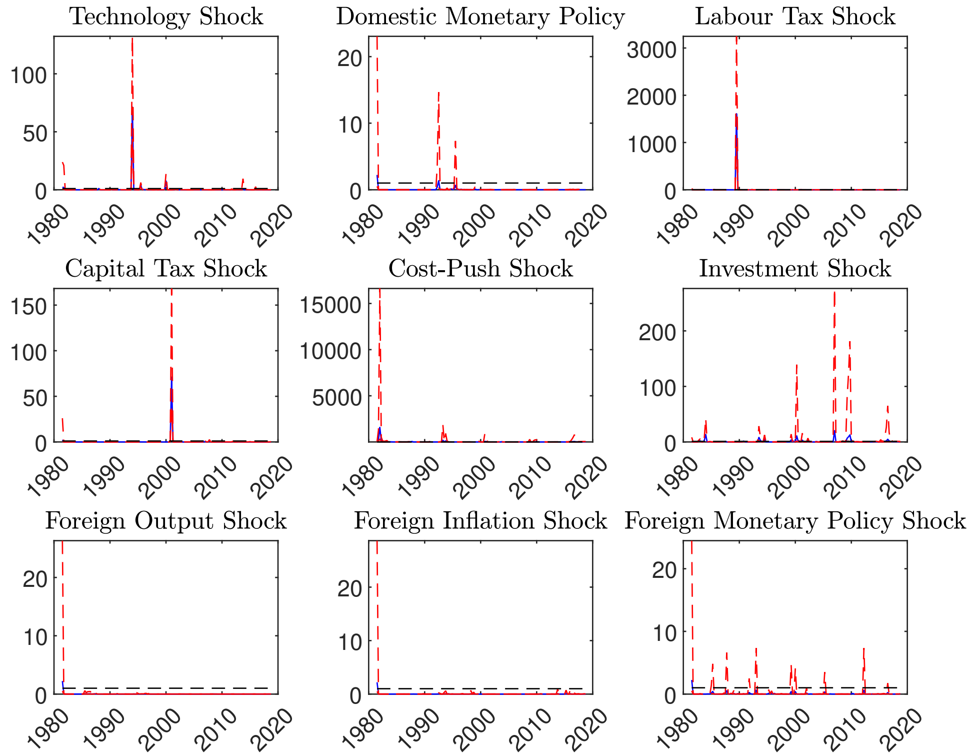

Figure 12: Median relative shares of stochastic-volatility in structural shocks (blue, but not so visually apparent) are the same as those plotted in Figure 4 in the paper. Here, we also show the estimated 16th and 84th percentiles of the distributions (i.e., the credible set) in dashed-red. The dashed-black line is the reference point of unity. As can be seen, the upper bound on the credible set is quite large and swamps the scale of the median estimates. In the paper we do not display the credible set.

	Technology	Monetary Policy	Labour Tax	Capital Tax	Cost-Push	Investment	Output*	Inflation*	Monetary Policy*
Median	0.0015	0.0008	0.0012	0.0010	1.6799	0.1004	0.0015	0.0008	0.0019
Mean	0.6100	0.0363	10.7872	0.5108	21.3007	0.9187	0.0188	0.0195	0.0510
Mode	0.0000	0.0001	0.0000	0.0001	0.1158	0.0093	0.0003	0.0001	0.0000
Skew	11.8536	8.1797	12.1245	12.1001	11.1304	4.6652	12.0901	12.0380	8.1916
Min	0.0000	0.0001	0.0000	0.0001	0.1158	0.0093	0.0003	0.0001	0.0000
Max	0.0713	0.0022	1.6092	0.0710	1.5780	0.0203	0.0022	0.0022	0.0022

Table 3: Descriptive statistics of the relative shares of stochastic-volatility in structural shocks over the sample period. Each column refers to the name of the respective structural shocks in the model. The asterisked names refer to Foreign variables. Note that the Max statistic was divided by 1000.

	Technology	Monetary Policy	Labour Tax	Capital Tax	Cost-Push	Investment	Output*	Inflation*	Monetary Policy*
h=1	0.3114	0.3575	0.0859	0.1288	0.0240	0.0906	0.0015	0.0000	0.0002
h=4	0.4664	0.2756	0.0845	0.0838	0.0186	0.0701	0.0009	0.0002	0.0001
h=8	0.5622	0.2295	0.0728	0.0598	0.0157	0.0591	0.0005	0.0004	0.0000
h=20	0.5954	0.2168	0.0664	0.0495	0.0149	0.0562	0.0004	0.0004	0.0000
h=40	0.5970	0.2198	0.0643	0.0462	0.0151	0.0568	0.0004	0.0004	0.0000
h=40	0.5967	0.2202	0.0642	0.0460	0.0151	0.0569	0.0004	0.0004	0.0000

Table 4: Average values of the forecast error variance decompositions over the sample period. Each column refers to the name of the respective structural shocks in the model. The asterisked names refer to Foreign variables.